Решења задатака са III колоквијума из Математике 1 30. 1. 2014.

1. Израчунати интеграле:

а) $\int \frac{\sin x}{\sqrt{2+\cos x}}\,dx$ б) $\int x e^x\,dx$.

Р. а) $\int \frac{\sin x}{\sqrt{2+\cos x}}\,dx = \left| \begin{array}{l} t = 2+\cos x \leftarrow \text{смена} \\ dt = (2+\cos x)'dx = -\sin x\,dx \end{array} \right| =$

$= \int \frac{-dt}{\sqrt{t}} = -\int t^{-\frac{1}{2}}dt = -\frac{1}{-\frac{1}{2}+1}t^{-\frac{1}{2}+1} = -2\sqrt{t} =$

$= -2\sqrt{2+\cos x} + C$.

б) $\int x e^x dx = /\text{парцијална интеграција}/ =$

$= \left| \begin{array}{l} u = x \to du = dx \\ dv = e^x dx \to v = \int e^x dx = e^x \end{array} \right| = x\cdot e^x - \int e^x dx =$

$= x\cdot e^x - e^x = (x-1)e^x + C$

2. Изр. интеграл $\int \frac{2x-1}{x^2-2x+3}dx$.

Р. $\int \frac{2x-1}{x^2-2x+3}dx = \int \frac{2x-1}{(x-1)^2+2}dx = \left| \begin{array}{l} t = x-1 \to x = t+1 \\ dt = dx \end{array} \right| =$

$= \int \frac{2(t+1)-1}{t^2+2}dt = \int \frac{2t+1}{t^2+2}dt = \int \frac{2t}{t^2+2}dt + \int \frac{dt}{t^2+2}$.

$\int \frac{2t}{t^2+2}dt = \left| \begin{array}{l} u = t^2+2 \\ du = 2t\,dt \end{array} \right| = \int \frac{du}{u} = \ln|u| = \ln(t^2+2) =$

$= \ln((x-1)^2+2) = \ln(x^2-2x+3)$

$\int \frac{dt}{t^2+2} = \int \frac{dt}{2\left(\left(\frac{t}{\sqrt{2}}\right)^2+1\right)} = \left| \begin{array}{l} u = \frac{t}{\sqrt{2}} \\ du = \frac{dt}{\sqrt{2}} \Rightarrow dt = \sqrt{2}\,du \end{array} \right| =$

$= \int \frac{\sqrt{2}\,du}{2(u^2+1)} = \frac{1}{\sqrt{2}}\int \frac{du}{u^2+1} = \frac{1}{\sqrt{2}}\operatorname{arctg} u = \frac{1}{\sqrt{2}}\operatorname{arctg}\left(\frac{t}{\sqrt{2}}\right) =$

$= \frac{1}{\sqrt{2}}\operatorname{arctg}\left(\frac{x-1}{\sqrt{2}}\right)$

Добијамо $\displaystyle\int \frac{2x-1}{x^2-2x+3}\,dx = \ln(x^2-2x+3) + \frac{1}{\sqrt{2}}\operatorname{arctg}\left(\frac{x-1}{\sqrt{2}}\right) + C.$

3. Наћи површину области ограничене кривама $f(x) = x^2+x-2$ и $g(x) = -x^2-3x+4$.

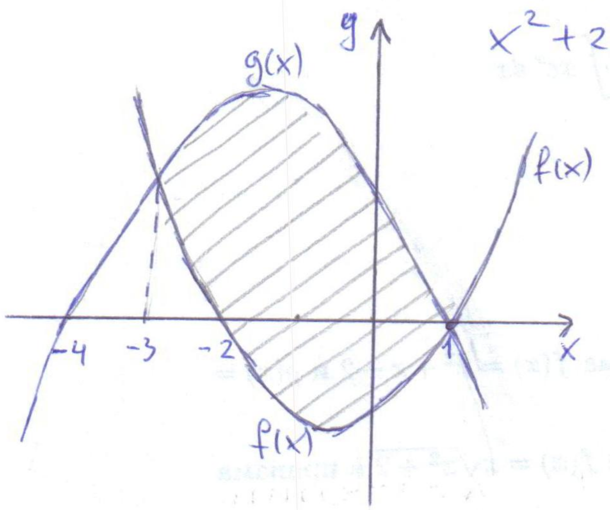
Р. $f(x) = x^2+x-2 = 0 \implies x_1=-2, \quad x_2 = 1$

$g(x) = -x^2-3x+4 = 0 \implies x_1 = -4, \quad x_2 = 1$

$f(x) = g(x) \implies x^2+x-2 = -x^2-3x+4$

$2x^2+4x-6 = 0 \quad /:2$

$x^2+2x-3 = 0 \implies \boxed{x_1=-3, \quad x_2=1}$




Ф-је $f(x)$ и $g(x)$ се секу у тачкама чије су x-координате $x_1=-3$ и $x_2=1$. Оне представљају границе интеграције. Са слике се види да је $g(x)$ горња, а $f(x)$ доња ф-ја. Због тога је

$$P = \int_{-3}^{1}(g(x)-f(x))\,dx = \int_{-3}^{1}\left((-x^2-3x+4)-(x^2+x-2)\right)dx =$$

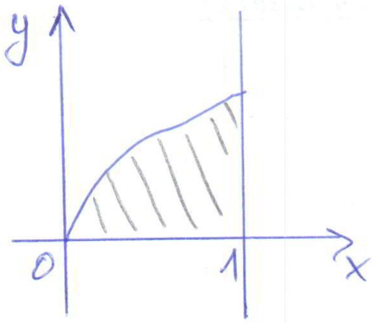
$$= \int_{-3}^{1}(-2x^2-4x+6)\,dx = \left(-2\frac{x^3}{3} - \cancel{4}^{2}\cdot\frac{x^2}{\cancel{2}_1} + 6x\right)\Big|_{-3}^{1} =$$

$$= -2\cdot\frac{1^3}{3} - 2\cdot 1^2 + 6\cdot 1 - \left(-2\cdot\frac{(-3)^3}{3} - 2\cdot(-3)^2 + 6\cdot(-3)\right) =$$

$$= -\frac{2}{3} + 4 - (18-18-18) = -\frac{2}{3} + 22 = \boxed{\frac{64}{3} = P}$$

4. Naći površinu oblasti ograničene krivom $f(x) = x\sqrt{x^2+2}$ i pravama $x=0$, $x=1$ i $y=0$.

P) $x \in [0,1] \Rightarrow x \geq 0 \Rightarrow f(x) = x\sqrt{x^2+2} \geq 0.$



$$P = \int_0^1 x\sqrt{x^2+2}\,dx =$$

$$\left| \begin{array}{l} t = x^2+2 \\ dt = (x^2+2)'dx = 2x\,dx \\ x\,dx = \frac{1}{2}dt \\ x = 1 \Rightarrow t = 1^2+2 = 3 \\ x = 0 \Rightarrow t = 0^2+2 = 2 \end{array} \right| =$$

$$= \int_0^1 \sqrt{x^2+2}\cdot x\,dx = \int_2^3 \sqrt{t}\cdot\frac{1}{2}dt = \frac{1}{2}\int_2^3 t^{\frac{1}{2}}dt =$$

$$= \frac{1}{2}\cdot\frac{1}{\frac{1}{2}+1}t^{\frac{1}{2}+1}\Big|_2^3 = \frac{1}{\cancel{2}}\cdot\frac{1}{\frac{3}{\cancel{2}}}t^{\frac{3}{2}}\Big|_2^3 = \frac{1}{3}\sqrt{t^3}\Big|_2^3 =$$

$$= \frac{1}{3}\sqrt{3^3} - \frac{1}{3}\sqrt{2^3} = \frac{1}{3}\cdot 3\cdot\sqrt{3} - \frac{1}{3}\cdot 2\sqrt{2} = \sqrt{3} - \frac{2}{3}\sqrt{2}.$$