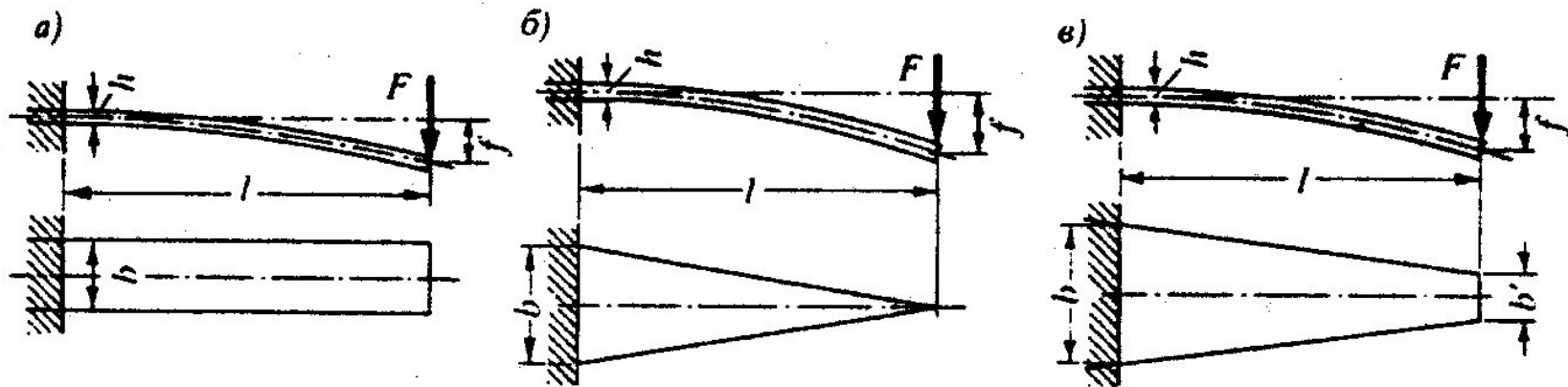


Proračun lisnatih opruga

Proračun lisnatih opruga sastoji se u proveru napona savijanja u uklještenju opruge:

$$\sigma_f = \frac{M}{W} = \frac{6Fl}{bh^2} \leq \sigma_{fdoz}$$

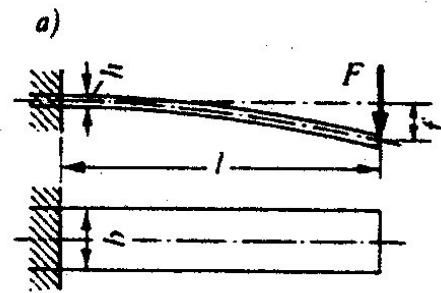
odakle sledi maksimalno opterećenje opruge: $F_{\max} = \frac{bh^2}{6l} \sigma_{fdoz}$



Proračun lisnatih opruga

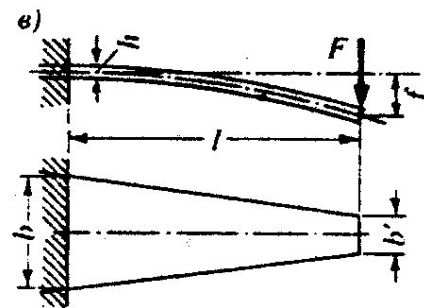
Deformacija opruge, tj. ugib na kraju konzole za lisnatu oprugu konstantnog poprečnog preseka iznosi:

$$f = \frac{Fl^3}{3EI} = 4 \frac{l^3}{bh^3} \frac{F}{E}$$



Deformacija opruge, tj. ugib na kraju konzole za oprugu trapeznog poprečnog preseka iznosi:

$$f = 4 \frac{3}{2 + \frac{b'}{b}} \frac{l^3}{bh^3} \frac{F}{E}$$

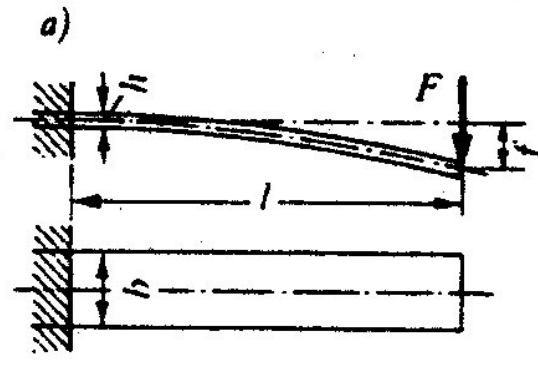


Proračun lisnatih opruga

Maksimalni ugibi opruga dobijaju se za opterećenje F_{\max} i iznose:

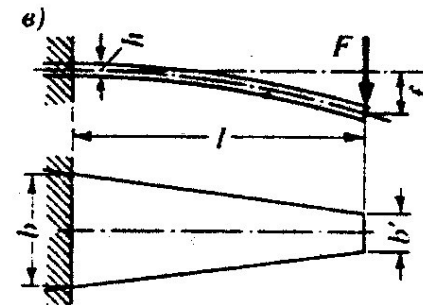
- za oprugu konstantnog poprečnog preseka

$$f_{\max} = \frac{2 l^2 \sigma_{fdoz}}{3 h E}$$



- za oprugu trapeznog poprečnog preseka

$$f_{\max} = \frac{2}{3} \frac{3}{2 + \frac{b'}{b}} \frac{l^2 \sigma_{fdoz}}{h E}$$



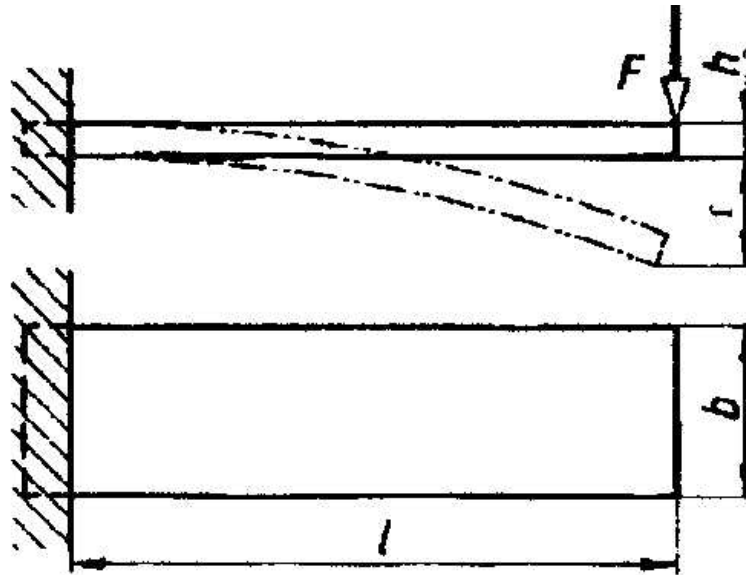
Proračun lisnatih opruga

Maksimalni deformacioni rad lisnatih opruga iznosi:

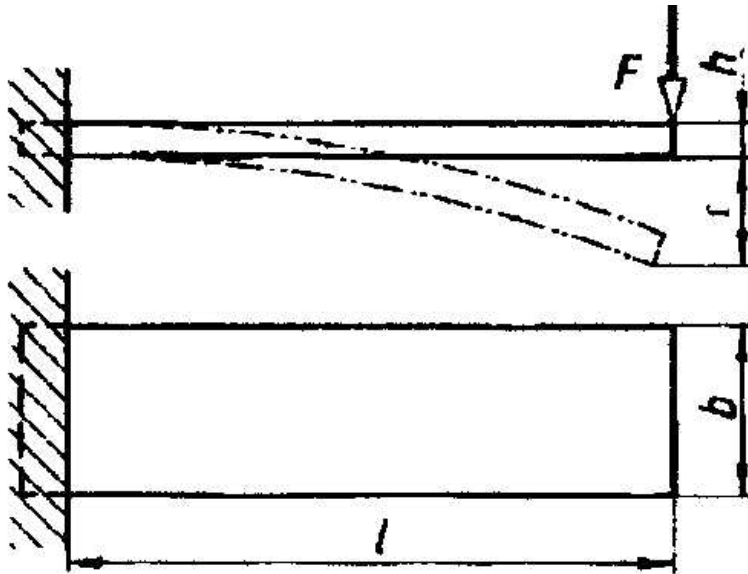
$$A_{\max} = \frac{F_{\max} f_{\max}}{2}$$

Proračun lisnatih opruga

Primer: Konzolno učvršćena pravougaona opruga od **Č4230** dužine **$l = 500\text{mm}$** , širine **$b = 60\text{mm}$** i debljine **$h = 5\text{mm}$** statički je opterećena na slobodnom kraju silom **$F = 200\text{kN}$** . Treba odrediti maksimalni ugib i stvarni napon od savijanja na mestu uklještenja, ako se računa sa najmanjom zateznom čvrstoćom materijala opruge.

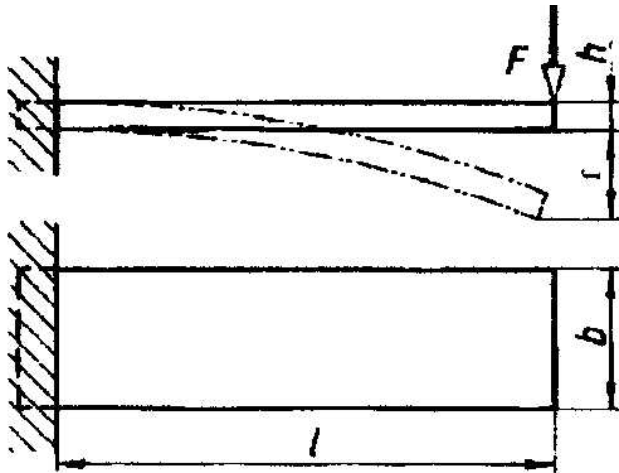


Proračun lisnatih opruga



$$f = \frac{Fl^3}{3EI} = 4 \frac{l^3}{bh^3} \frac{F}{E} = 4 \frac{500^3 \text{ mm}^3}{60 \text{ mm} \cdot 5^3 \text{ mm}^3} \frac{200 \text{ N}}{206000 \frac{\text{N}}{\text{mm}^2}} = 64.72 \text{ mm}$$

Proračun lisnatih opruga



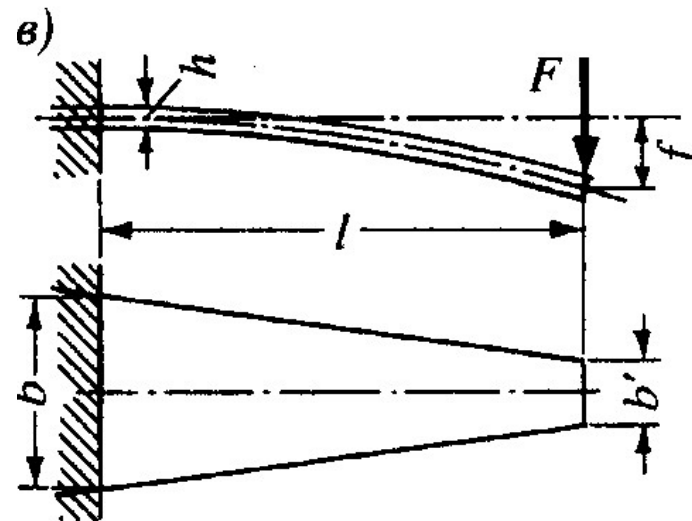
Č4230 ► $R_m = 1500 \dots 1700 \text{ N/mm}^2$

$$\sigma_{fdoz} = 0.7R_m = 0.7 \cdot 1500 \frac{\text{N}}{\text{mm}^2} = 1050 \frac{\text{N}}{\text{mm}^2}$$

$$\sigma_f = \frac{M}{W} = \frac{6Fl}{bh^2} = \frac{6 \cdot 200\text{N} \cdot 500\text{mm}}{60\text{mm} \cdot 5^2\text{mm}^2} = 400 \frac{\text{N}}{\text{mm}^2} < \sigma_{fdoz}$$

Proračun lisnatih opruga

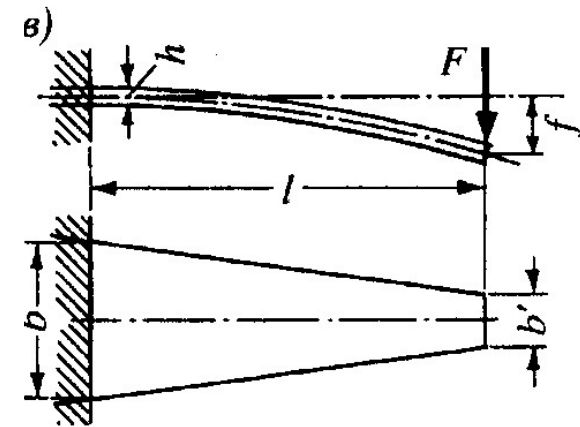
Primer: Odrediti dimenzije konzolno učvršćene trapezne lisnate opruge, debljine $h = 5\text{mm}$ i odnosa širina $b/b' = 0.3$, napravljene od Č4230, koja je statički opterećena na slobodnom kraju silom $F = 200\text{kN}$. Ugib opruge iznosi $f = 66.7\text{mm}$, dok je napon od savijanja na mestu ukleštenja $\sigma_f = 400\text{ N/mm}^2$.



Proračun lisnatih opruga

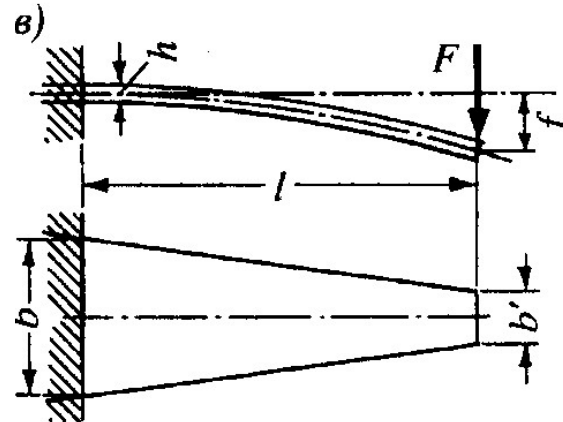
$$f = \frac{2}{3} \frac{3}{2 + \frac{b'}{b}} \frac{l^2}{h} \frac{\sigma_f}{E} \Rightarrow l = \sqrt{\frac{f \cdot h \cdot E}{\frac{2}{3} \frac{3}{2 + \frac{b'}{b}} \sigma_f}}$$

$$l = \sqrt{\frac{66.7 \text{ mm} \cdot 5 \text{ mm} \cdot 206000 \frac{\text{N}}{\text{mm}^2}}{\frac{2}{3} \frac{3}{2 + 0.3} 400 \frac{\text{N}}{\text{mm}^2}}} = 443.3 \text{ mm}$$



Usv. $l = 450 \text{ mm}$

Proračun lisnatih opruga



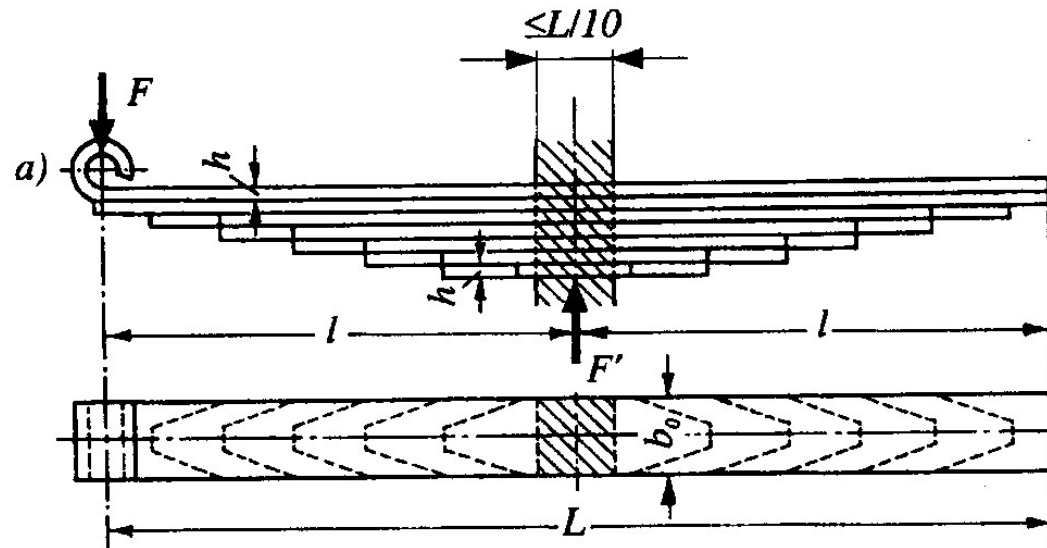
$$\sigma_f = \frac{6Fl}{bh^2} \Rightarrow b = \frac{6Fl}{\sigma_f h^2} = \frac{6 \cdot 200\text{N} \cdot 450\text{mm}}{400 \frac{\text{N}}{\text{mm}^2} 5^2 \text{mm}^2} = 54\text{mm}$$

Usv. $b = 55\text{mm}$

$$\frac{b'}{b} = 0.3 \Rightarrow b' = 0.3b = 0.3 \cdot 55\text{mm} = 16.5\text{mm}$$

Proračun lisnatih opruga

Primer: Transportna prikolica sa jednom osovinom oslanja se na 2 gibanja. Nosivost prikolice je $1.5t$. Gibanj se sastoji od $z = 5$ listova, od čega su $z' = 2$ iste dužine. Dimenzije gibanja su: $b_0 = 50\text{mm}$, $h = 7\text{mm}$ i $l = 345\text{mm}$. Proveriti normalni napon od savijanja i odrediti krutost gibanja, čiji je materijal Č2134.



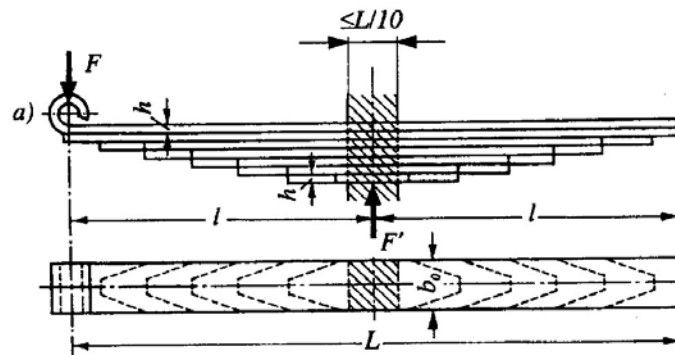
Proračun lisnatih opruga

$$\checkmark 2134 \blacktriangleright R_m = 1400 \dots 1600 \text{ N/mm}^2$$

$$\sigma_{\text{fdoz}} = 0.5R_m = 0.5 \cdot 1400 \frac{\text{N}}{\text{mm}^2} = 700 \frac{\text{N}}{\text{mm}^2}$$

$$F_1 = \frac{Q}{2 \cdot 2} = \frac{1500 \text{ kg} \cdot 9.81 \frac{\text{m}}{\text{s}^2}}{4} = 3678.75 \text{ N}$$

$$\sigma_f = \frac{6F_1 l}{z \cdot b_0 \cdot h^2} = \frac{6 \cdot 3678.75 \text{ N} \cdot 345 \text{ mm}}{5 \cdot 50 \text{ mm} \cdot 7^2 \text{ mm}^2} = 621.6 \frac{\text{N}}{\text{mm}^2} < \sigma_{\text{fdoz}}$$

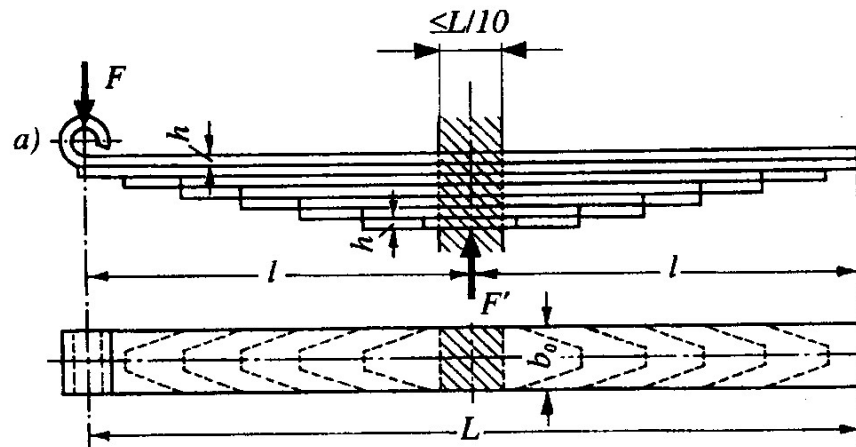


Proračun lisnatih opruga

$$b = zb_0 = 5 \cdot 50\text{mm} = 250\text{mm}$$

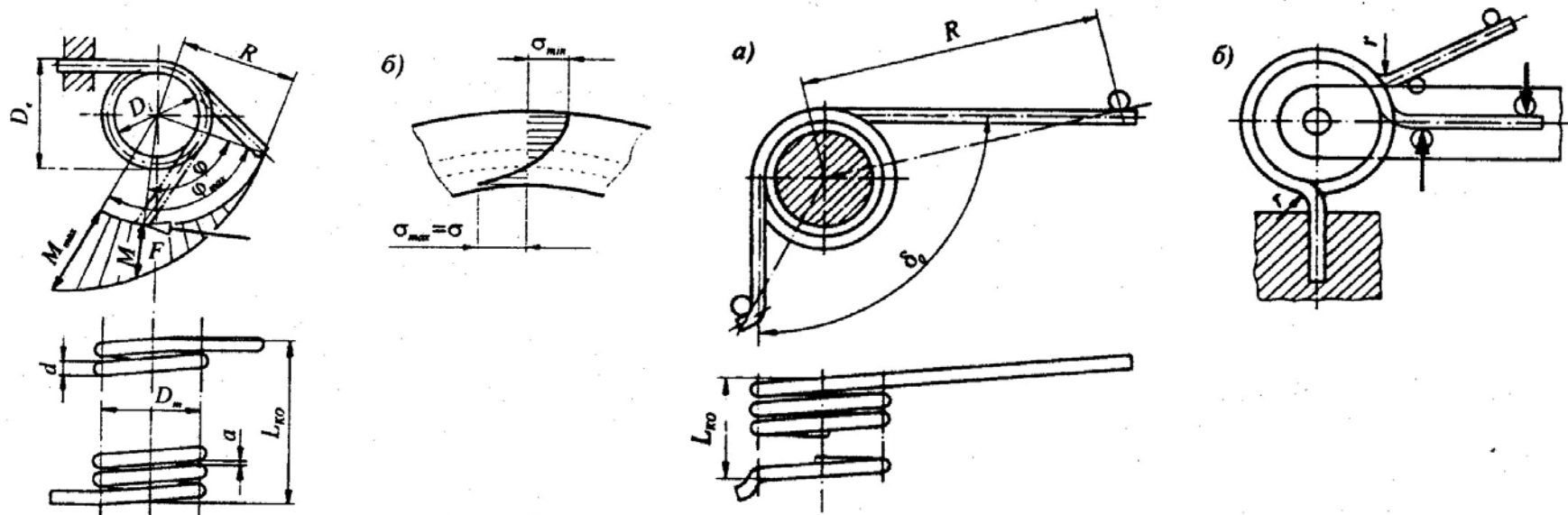
$$f = 4 \frac{3}{2 + \frac{z'}{z}} \frac{l^3}{bh^3} \frac{F_1}{E} = 4 \frac{3}{2 + \frac{2}{5}} \frac{345^3 \text{mm}^3}{250\text{mm} \cdot 7^3 \text{mm}^3} \frac{3678.75\text{N}}{206000 \frac{\text{N}}{\text{mm}^2}} = 42.76\text{mm}$$

$$c = \frac{F_1}{f} = \frac{3678.75\text{N}}{42.76\text{mm}} = 86.03 \frac{\text{N}}{\text{mm}}$$



Proračun uvrtnih zavojnih opruga

Proračun nosivosti uvrtnih zavojnih opruga sastoji se u proveru napona savijanja u poprečnom preseku žice.

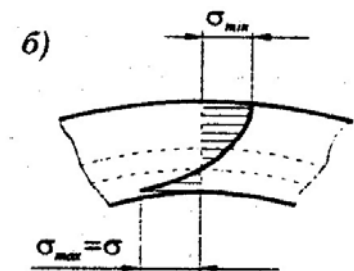
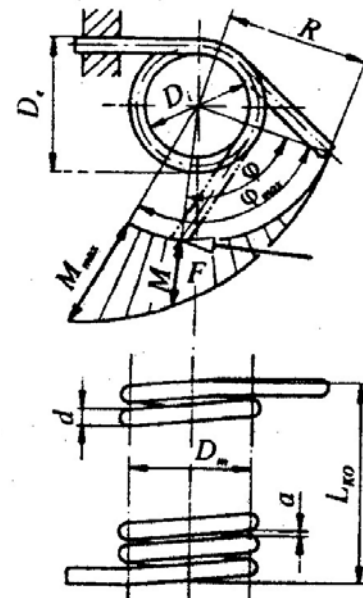


Proračun uvrtnih zavojnih opruga

Ako na slobodan krak opruge dužine R deluje sila F , onda je opruga opterećena momentom $M=FR$, pa je napon savijanja na unutrašnjoj strani zavoja:

$$\sigma_f = k_w \frac{M}{W} = k_w \frac{32FR}{d^3 \pi} \leq \sigma_{fdoz}$$

k_w - faktor koncentracije napona na unutrašnjoj strani zavoja opruge



Proračun uvrtnih zavojnih opruga

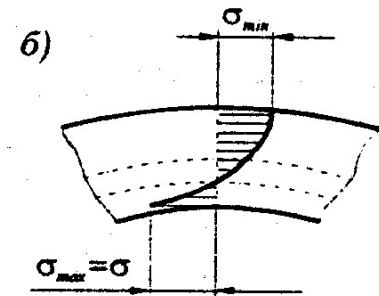
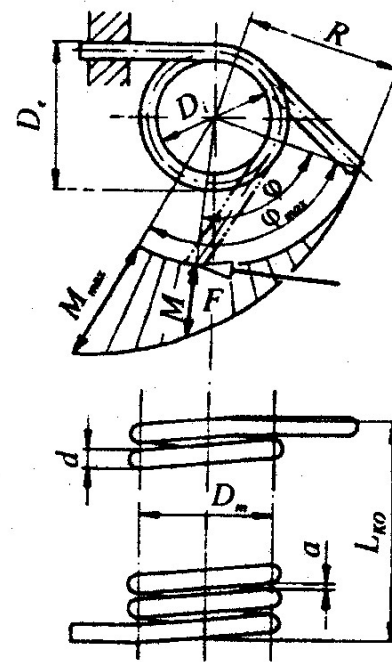
Ugao deformisanja uvrtnje zavojne opruge:

$$\varphi = \frac{ML}{EI} = \frac{MD_m \pi n}{EI}$$

Krutost uvrtnje zavojne opruge:

$$c_\varphi = \frac{M}{\varphi} = \frac{EI}{L}$$

$$I = \frac{d^4 \pi}{64}$$



Proračun uvrtnih zavojnih opruga

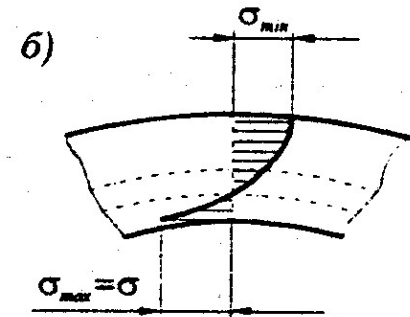
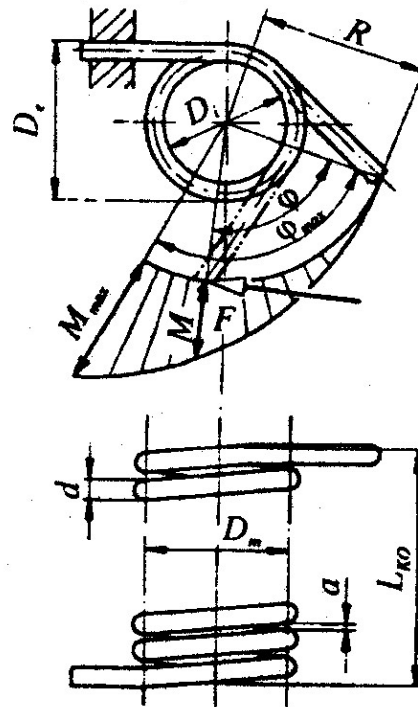
Potreban prečnik žice za izradu opruge:

$$d = k_1 \frac{\sqrt[3]{M}}{1 - k_2}$$

$$k_1 \approx 0.22 \quad \text{za} \quad d < 5\text{mm}$$

$$k_1 \approx 0.24 \quad \text{za} \quad d \geq 5\text{mm}$$

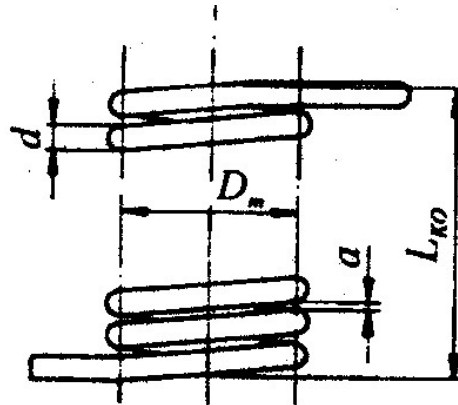
$$k_2 \approx 0.06 \frac{\sqrt[3]{M}}{d_t}$$



Proračun uvrtnih zavojnih opruga

Dužina neopterećene uvrtnje zavojne opruge:

$$L_{K0} = n(a + d) + d$$



Potrebna dužina žice neopterećene uvrtnje zavojne opruge:

$$d + a \leq \frac{D_m}{4} \Rightarrow L = D_m \pi n$$

$$d + a > \frac{D_m}{4} \Rightarrow L = n \sqrt{(D_m \pi)^2 + (a + d)^2}$$

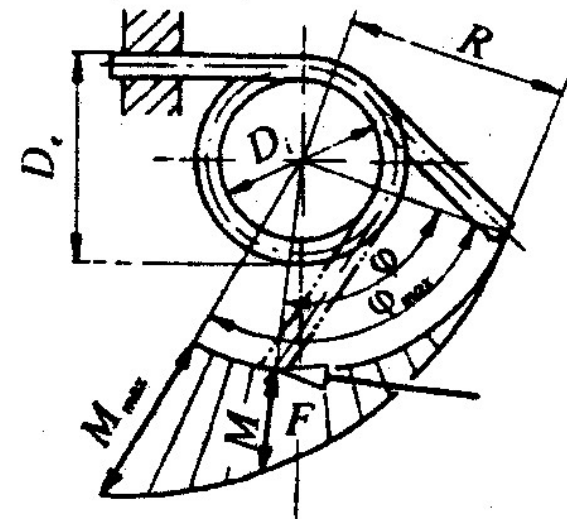
Proračun uvrtnih zavojnih opruga

Srednji prečnik opruge pri deformacionom uglu φ :

$$D_{m\varphi} = D_m \frac{2\pi n}{2\pi n + \varphi}$$

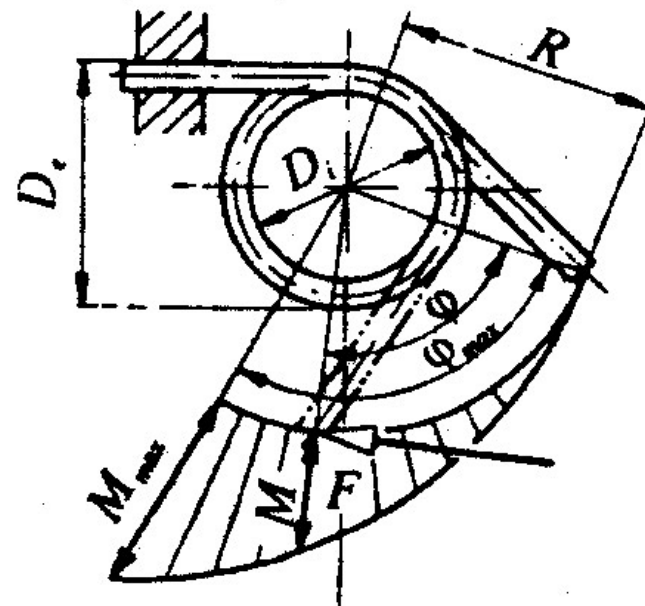
Unutrašnji prečnik opruge pri deformacionom uglu φ ne sme biti manji od prečnika trna:

$$D_{u\varphi} = D_{m\varphi} - d = D_m \frac{2\pi n}{2\pi n + \varphi} - d \geq d_t$$



Proračun uvrtnih zavojnih opruga

Primer: Dimenzionisati uvrtnu zavojnu oprugu koja prenosi maksimalnu silu $F = 300\text{N}$ preko tangentno izvedenih krajeva dužine $R = 50\text{mm}$ i deformacionim uglom $\varphi_{\max} = 120^\circ$. Opruga je pretežno statički opterećena i postavljena na osovinicu $d_t = 30\text{mm}$ sa $a = 1\text{mm}$.



Proračun uvrtnih zavojnih opruga

$$d_t = (0.8 \div 0.9)D_u \Rightarrow D_u = \frac{d_t}{(0.8 \div 0.9)} = 33 \div 37[\text{mm}]$$

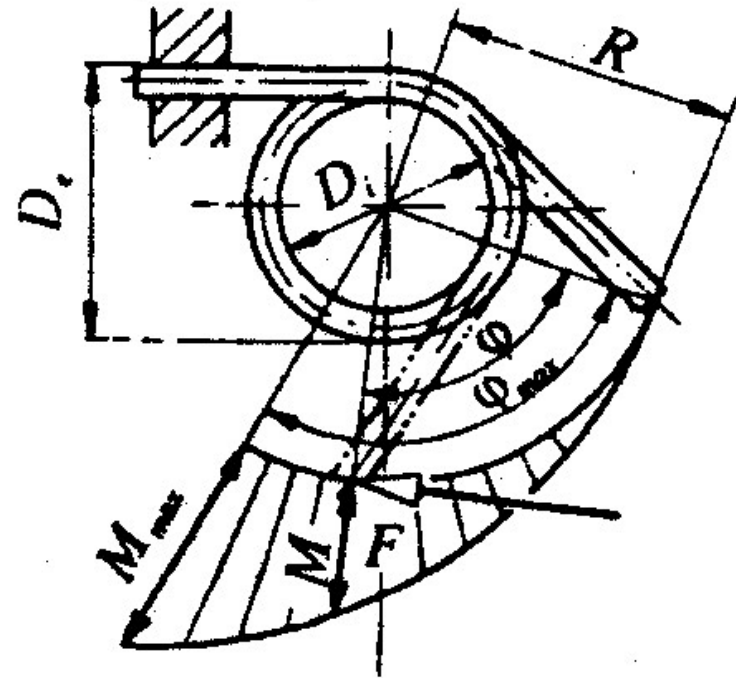
$$\Rightarrow \text{usv. } D_u = 34\text{mm}$$

$$M = FR = 15000\text{Nmm}$$

$$d = k_1 \frac{\sqrt[3]{M}}{1 - k_2}$$

$$\text{usv. } k_1 \approx 0.24 \quad \text{za } d \geq 5\text{mm}$$

$$k_2 \approx 0.06 \frac{\sqrt[3]{M}}{d_t} = 0.0435$$



Proračun uvrtnih zavojnih opruga

$$d = k_1 \frac{\sqrt[3]{M}}{1 - k_2} = 6.18 \text{mm}$$

Таблица П24-5а Минималне вредности затезне чврстоће у N/mm² округле жице за опруге према DIN 17223 T1

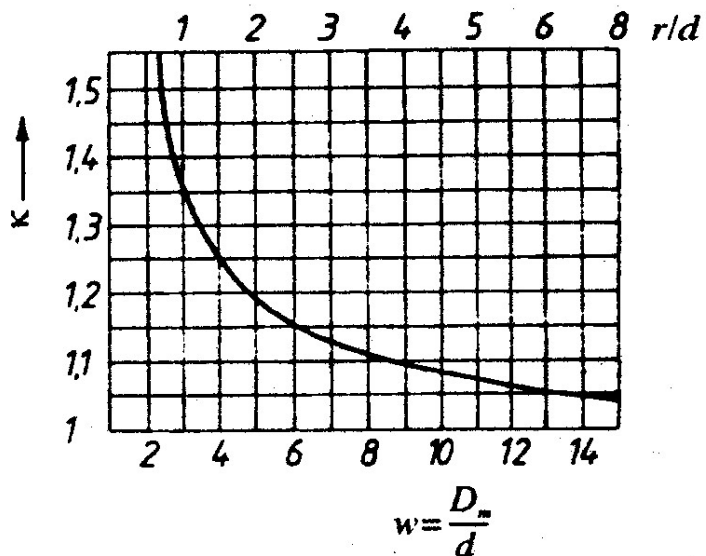
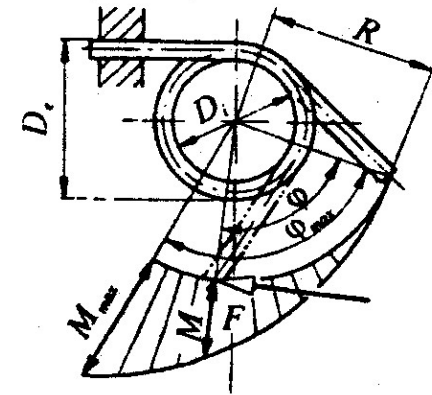
d mm	Квалитет жице						d mm	Квалитет жице					
	A	B	C	D	FD	VD		A	B	C	D	FD	VD
0,30		2370		2640			2,50	1460	1690	1900	1900	1570	1471
0,32		2350		2640			2,60	1450	1670	1890	1890	1570	1471
0,34		2330		2610			2,80	1420	1650	1860	1860	1570	1471
0,36		2310		2590			3,00	1410	1630	1840	1840	1521	1432
0,38		2290		2570			3,20	1390	1610	1820	1820	1521	1432
0,40		2270		2560			3,40	1370	1590	1790	1790	1521	1432
0,43		2250		2530			3,60	1350	1570	1770	1770	1481	1403
0,45		2240		2510			3,80	1340	1550	1750	1750	1481	1403
0,48		2220		2490			4,00	1320	1530	1740	1740	1481	1403
0,50		2200		2480			4,25	1310	1510	1710	1710	1442	1373
0,53		2180		2460			4,50	1290	1500	1690	1690	1442	1373
0,56		2170		2440			4,75	1270	1480	1680	1680	1442	1373
0,60		2140		2410			5,00	1260	1460	1660	1660	1403	1344
0,63		2130		2390			5,30	1240	1440	1640	1640	1403	1344
0,65		2120		2380			5,60	1230	1430	1620	1620	1403	1344
0,70		2090		2360			6,00	1210	1400	1590	1590	1403	1344
0,75		2070		2330			6,30	1190	1390	1570	1570	1403	1344
0,80		2050		2310			6,50	1180	1380	1560	1560	1364	1305
0,85		2030		2290			7,00	1160	1350	1540	1540	1364	1305
0,90		2010		2270			7,50	1140	1330	1510	1510	1364	1305
0,95		2000		2250			8,00	1120	1310	1490	1490	1295	
1,00	1720	1980		2230	1766	1668	8,50	1110	1290	1470	1470	1295	
1,05	1710	1960		2210	1766	1668	9,00	1090	1270	1450	1450	1295	
1,10	1690	1950		2200	1766	1668	9,50	1070	1260	1430	1430	1295	
1,20	1670	1920		2170	1717	1619	10,00	1060	1240	1410	1410	1256	
1,25	1660	1910		2150	1717	1619	10,50		1220	1390	1390	1256	
1,30	1640	1900		2140	1717	1619	11,00		1210	1380	1380	1256	
1,40	1620	1870		2110	1717	1619	12,00		1180	1350	1350	1256	
1,50	1600	1850		2090	1668	1570	12,50		1170	1330	1330	1256	
1,60	1590	1830		2060	1668	1570	13,00		1160	1320	1320	1256	
1,70	1570	1810		2040	1668	1570	14,00		1130	1290	1290	1256	
1,80	1550	1790		2020	1668	1570	15,00		1110	1270	1270		
1,90	1540	1770		2000	1668	1570	16,00		1090	1240	1240		
2,00	1520	1760	1980	1980	1619	1521	17,00		1070	1220	1220		
2,10	1510	1740	1970	1970	1619	1521	18,00		1050	1200	1200		
2,25	1490	1720	1940	1940	1619	1521	19,00		1030	1180	1180		
2,40	1470	1700	1920	1920	1619	1521	20,00		1020	1160	1160		

► usv. vučenu žicu za opruge kvaliteta **B** i prečnika **d = 6mm**. 22

Proračun uvrtnih zavojnih opruga

$$\sigma_f = k_w \frac{M}{W} = k_w \frac{32FR}{d^3 \pi} \leq \sigma_{fdoz}$$

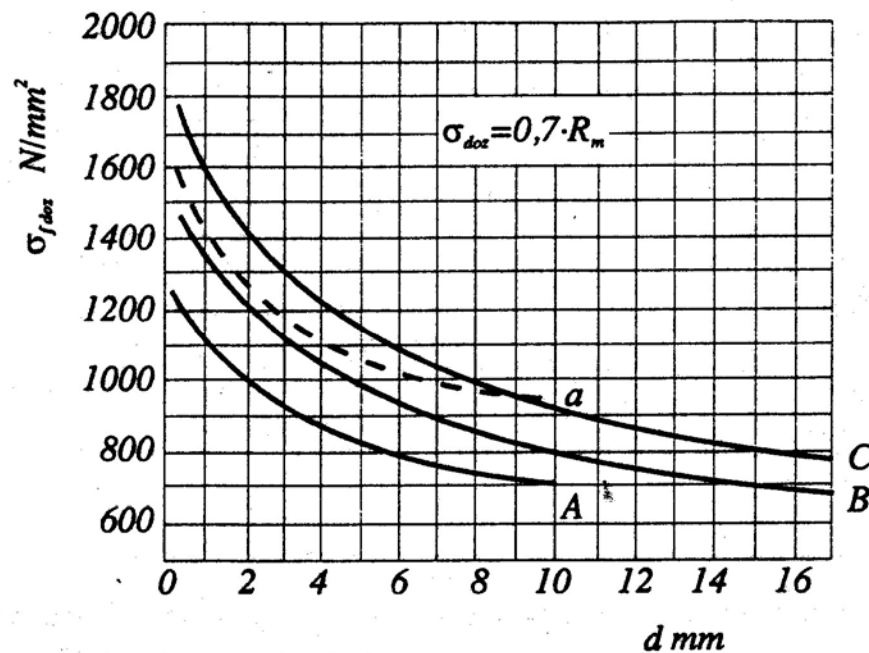
$$D_m = D_u + d = 40\text{mm}$$



$$w = \frac{D_m}{d} = 6.67 \Rightarrow k_w = 1.14$$

Proračun uvrtnih zavojnih opruga

$$\sigma_f = k_w \frac{32FR}{d^3 \pi} = 806.4 \frac{\text{N}}{\text{mm}^2}$$



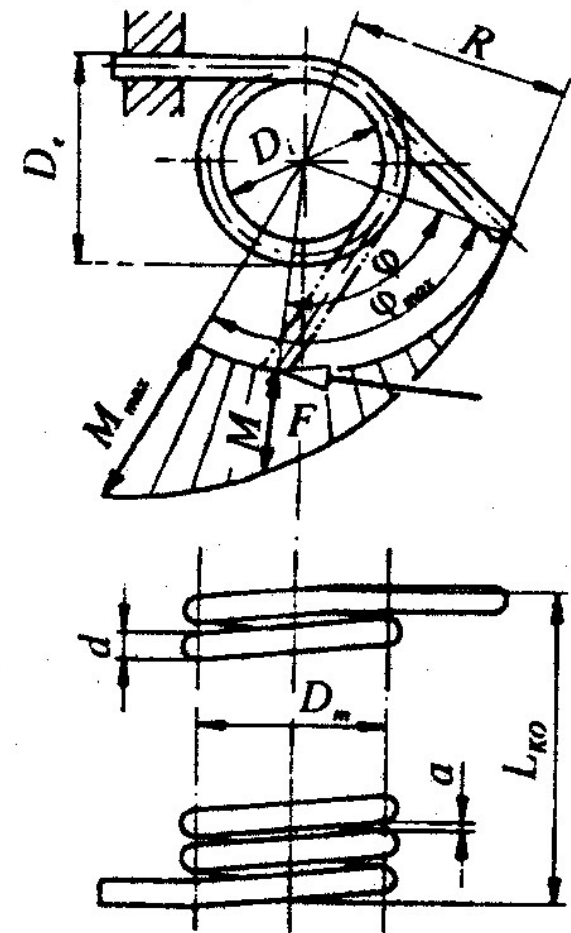
za vučenu žicu za opruge kvaliteta **B** i prečnika $d = 6\text{mm}$:

$$\blacktriangleright \sigma_{fdoz} = 940 \text{ N/mm}^2 \quad \Rightarrow \quad \sigma_f < \sigma_{fdoz}$$

Proračun uvrtnih zavojnih opruga

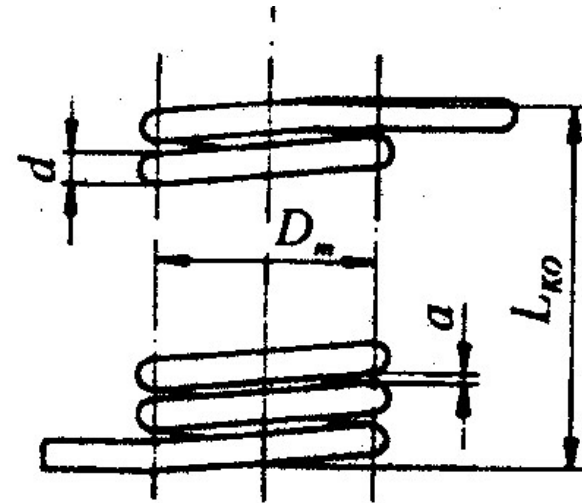
$$I = \frac{d^4 \pi}{64} = 63.6 \text{mm}^4$$

$$\varphi = \frac{ML}{EI} = \frac{MD_m \pi n}{EI} \Rightarrow n = \frac{\varphi EI}{MD_m \pi} \approx 15$$



Proračun uvrtnih zavojnih opruga

$$L_{K0} = n(a + d) + d = 111\text{mm}$$



$$d + a = 7\text{mm} \leq \frac{D_m}{4} = 10\text{mm} \Rightarrow L = D_m \pi n + 2R = 1985\text{mm}$$

$$D_{u\varphi} = D_m \frac{2\pi n}{2\pi n + \varphi} - d = 33.13\text{mm} \geq d_t = 30\text{mm}$$