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Leonhard Euler (1707-1783) and Rigid Body Dynamics

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The introductory test of this volume is dedicated to the 300th anniversary of the birth of Leonhard Euler, one of the most distinguished scientists of the 18th century in the fields of mathematics and mechanics. Born on 15 April 1707 in Basel, Switzerland, he devoted his life to mathematics instead to theology, much against his father wish and owing to famous Bernoulli. Although the majority of his work concerned elementary mathematics, he also contributed significantly to astronomy and physics.
Partially blind in his late twenties and totally blind in his late years, Euler was nonetheless exceptionally productive in his studies – 500 titles appeared during his lifetime and 400 more were published after his death. Laplace used to tell his students: "Read Euler, read Euler! He is the teacher of us all!"

Introduction

A statement attributed to Pierre-Simon Laplace expresses Euler's influence on mathematics: "*Read Euler, read Euler, he is the teacher (master) of us all*".

The Euler Committee of the Swiss Academy of Sciences was founded in 1907 with the task to publish all scientific books, papers and the correspondence of Leonhard Euler (1707-1783) in a scientific edition.

Euler was one of the leading and most famous scientists in the mathematics as well as in the mechanics of the 18th century.

Euler's books and papers are edited in the Series I-III, the correspondence in the Series IV. In the last 90 years 71 volumes of the Series I-III have been published. The last three volumes are in preparation and should appear shortly. The Series IV with Euler's scientific correspondence will contain 10 volumes. Four volumes have been published and three volumes are in preparation.

Fugure 1. Aleksandr Mikhailovich Lyapunov (1857 – 1918) Editor of two volumes of *Euler's collected works*.

This huge endeavor can only succeed with the aid of internationally acclaimed scientists as coworkers and with the financial support of the Swiss National Science Foundation, the Swiss Academy of Sciences and many long-term substantial contributions from Swiss industrial corporations.

A. M. Lyapunov opened a new page in the history of global science. He also contributed as an editor of two volumes of ***Euler's collected works***. He took part in the publication of Euler's Selected Works and was the editor of the 18th and 19th part of this miscellany.

The "Euler phenomenon"

Three factors go a long way towards explaining the "Euler phenomenon": First of all, his - perhaps unique - gifted memory. He seemed to have remembered whatever he had heard, seen, thought, or written in his whole life, as countless contemporaries confirmed. For example, in his advanced age he was able to delight his relatives, friends, and acquaintances with a literal (Latin) recitation of any song from Virgil's Aeneis, and he could reproduce by heart the minutes of the academy meetings decades later, not mentioning his memory concerning mathematics. Furthermore, Euler's prodigious memory went hand in hand with a rare ability to concentrate. Noise and bustle in his immediate environment hardly disturbed his thinking: "A child on his knees, a cat on his back - this is how he wrote his immortal works," reported Thiébault, his colleague from the Berlin academy. The third factor in the "Euler mystery" is, quite simply, constant, meticulous work.

Reputation

Leonhard Euler's influence and reputation were already impressive during his lifetime. For almost two decades he was (according to Andreas Speiser) the intellectual leader of the Protestant part of Germany, and (according to Eduard Winter) he performed inestimable services as the "golden bridge between two academies". The 10 volumes of his correspondence testify to this role, as does the fact that, during his Berlin years, Euler published 109 papers in the

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Petersburger Kommentare and 119 papers in the Mémoires of the Berlin academy. And although Euler's energy was sufficient for him to keep up his activities at both institutions, the institutions themselves could not easily cope with the almost inexhaustible tide of Euler's productivity. To judge simply from the extent of his work, Euler is in the company of the most prolific members of the human race, e.g, Voltaire, Leibniz, Telemann or Goethe. The directory of Euler's writings published by G. Eneström (1910-1913) takes up an entire volume and contains almost 900 titles, some 40 books among them.

Productivity

The following table summarizes the extent of Euler's writings specified by him as ready for publication, arranged according to decades (not included are a few dozen works that have not yet been dated):

Year	Works	%
1725–1734	35	5
1735–1744	50	10
1745–1754	150	19
1755–1764	110	14
1765–1774	145	18
1775–1783	270	34

Specific disciplines

With respect to specific disciplines, the writings are classified approximately as follows:

- Algebra, number theory, analysis 40%
- Mechanics and other physics 8%
- Geometry, including trigonometry 18%
- Astronomy 11%
- Ship theory, artillery, architecture 2%
- Philosophy, music theory, theology, and anything else not included above 1%

The classification of Euler's purely mathematical works is approximately as follows:

- Algebra, combination and probability theory 10%
- Number theory 13%
- Fundamental analysis and differential calculus 7%
- Infinite series 13%
- Integral calculus 20%
- Differential equations 13%
- Calculus of variations 7%
- Geometry, including differential geometry 17%

Awards

Leonhard Euler won 12 international academy prizes, not counting the prizes of his sons Johann Albrecht (7) and Karl (1), which can essentially also be credited to Euler's account. The French King Louis XVI awarded Euler 1000 rubles for his "second ship theory", and the Russian empress Catherine II gave him double that amount so that the blind doyen of Petersburg could receive a supplementary salary in 1773.

Influence

As far as Euler is concerned, the opinions of the most important mathematicians are unanimous. Laplace used to say to his students: "Read Euler, read Euler! He is the master of us all!" and Gauss explained emphatically: "The study of Euler's works remains the best instruction in the various areas of mathematics and can be replaced by no other." Indeed, through his books, which are consistently characterized by the highest striving for clarity and simplicity and which represent the first actual textbooks in a modern sense, Euler became the premier teacher of Europe not only of his time but well into the 19th century.

Formation and Training

1707 Born on 15 April in Basel, the son of the Protestant minister Paul Euler and Margaretha Brucker.

Leonhard Euler's father was Paul Euler who studied theology at the University of Basel and attended Jacob Bernoulli's lectures there. In fact Paul Euler and Johann Bernoulli both lived in Jacob Bernoulli's house while they were undergraduates at Basel. Leonhard Euler was born in Basel, but the family moved to Riehen when he was one year old and it was in Riehen, not far from Basel, that Leonard was brought up. Paul Euler had some mathematical training and he was able to teach his son elementary mathematics along with other subjects.

Euler's father wanted his son to follow him into the church and sent him to the University of Basel to prepare for the ministry. He entered the University in 1720, at the age of 14, first to obtain a general education before going on to more advanced studies. Johann Bernoulli soon discovered Euler's great potential for mathematics in private tuition that Euler himself engineered.

In 1720 Leonhard entered Basel University, which was founded in 1460. Initially he studied theology, Oriental languages and history, but soon switched to mathematics under Johann Bernoulli (1667-1748), who became the world's most cited mathematician following the death of Isaac Newton (1643-1727). Quick to recognize Euler's mathematical genius, Bernoulli challenged him by having him read the works of the masters, and especially by instructing him personally in contemporary mathematical research.

In 1723 Euler completed his Master's degree in philosophy having compared and contrasted the philosophical ideas of Descartes and Newton. He began his study of theology in the autumn of 1723, following his father's wishes, but, although he was to be a devout Christian all his life, he could not find the enthusiasm for the study of theology, Greek and Hebrew that he found in mathematics. Euler obtained his father's consent to change to mathematics after Johann Bernoulli had used his persuasion. The fact that Euler's father had been a friend of Johann Bernoulli's in their undergraduate days undoubtedly made the task of persuasion much easier.

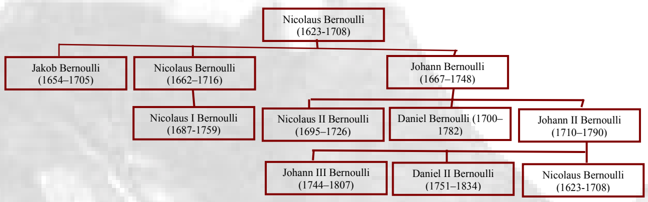


Figure 2. Family tree of the Bernoulli family

Euler completed his studies at the University of Basel in 1726. He studied many mathematical works during his time in Basel, and Calinger reconstructed many of the works that Euler read following the advice of Johann Bernoulli. They include works by Varignon, Descartes, Newton, Galileo, van Schooten, Jacob Bernoulli, Hermann, Taylor and Wallis. By 1726 Euler had already a paper in print, a short article on isochronous curves in a resisting medium. In

1727 he published another article on reciprocal trajectories and submitted an entry for the 1727 Grand Prize of the Paris Academy on the best arrangement of masts on a ship.

In 1727, Catherine I of Russia invited Euler to join the faculty of the Academy of Sciences in St. Petersburg. He became chairman of mathematics there in 1733, replacing Daniel Bernoulli. In 1735, he lost sight in one eye while working around the clock for three days to solve a mathematic problem that took other mathematicians months to solve. While in Russia, he prepared some 90 papers for publication and wrote the two-volume book Mechanics. He collaborated with Daniel Bernoulli in the field of fluid mechanics and derived the equation that related velocity and pressure, which became known as Bernoulli's equation. He also conceived of pressure as something that could change from point to point throughout a fluid.

Figure 3. Methodus inveniendi - Leonhard Euler
The cover page of Euler's Methodus inveniendi lineas curvas.

In 1741, at the urging of Frederick the Great, Euler moved to Berlin and became professor of mathematics at the Berlin Academy of Sciences, which he turned into a major academy. Over the next 25 years, Euler prepared at least 380 papers for publication. After his relationship with Frederick deteriorated, he accepted the invitation from Catherine the Great to return to St. Petersburg in 1766 where he became director of the Academy of Sciences. Soon after his return, he became almost totally blind. Nevertheless, he excelled at solving complex calculations in his head. While in St. Petersburg, he worked on developing a better theory of lunar motion that involved the interactions of the Sun, Moon, and Earth.

Euler contributed to the subjects of geometry, calculus, trigonometry, and number theory. He standardized mathematical notation using Greek symbols that continue to be used today. He also contributed to the fields of astronomy, mechanics, optics, and acoustics, and made a major contribution to theoretical aerodynamics. He derived the continuity equation and the equations for the motion of an in viscid, incompressible fluid.

Euler suffered a stroke and died on September 18, 1783 in St. Petersburg.

First Petersburg period

1727 Euler's thesis entitled De Sono (On Acoustics) formed the basis for his application for a post as professor of physics in Basel, but he was passed over on account of his youth. Through the help of the Bernoullis, he was offered a position in St. Petersburg at the Academy of Science, founded by Peter the Great in 1725. There he worked first as an assistant professor, then from 1730 as a professor and member of the academy (he had no teaching commitments, though he did write a textbook on elementary mathematics). The principal contributions of this early Petersburg period include a two-volume work on mechanics, a book on music theory and Scientia navalis (about hydrodynamics, shipbuilding and navigation), which was eventually published in 1749.

1734 At the beginning of January, Euler married Katharina Gsell, a daughter of a Swiss painter George Gsell, who was working in St. Petersburg. Euler's son Johann Albrecht was born at the end of November, the only one of his offspring to follow in his footsteps as a mathematician and member of the Academy. Only three of Euler's thirteen children would survive him. He had twenty-one grandchildren.

1738 As a result of a severe abscess, Euler lost the sight of his right eye.

Berlin years

1741 Conscious of the political turmoil in the Russian empire, Euler accepted Frederick II's offer of a professorship at a newly established Prussian Academy ("Berlin Academy") and settled with his family in Berlin. There he held a position as director of the mathematics department. Maupertuis, who in 1736 made a name for himself in a famous expedition to Lapland (the purpose of which was to determine whether the Earth was indeed an oblate spheroid) became president of the Academy, though as a scientist, he ranked far below Euler.

In addition to hundreds of treatises written during the Berlin period, Euler produced major works on the calculus of variations, the theory of special functions, differential equations, astronomy as well as a second masterpiece on mechanics and a popular work on physics and philosophy titled Lettres à une princesse d'Allemagne. The basic outline of his celebrated work on algebra also dates from the Berlin period. During this time, Euler maintained active connections with the Petersburg Academy, and he helped to promote interactions between the two internationally renowned academies. Euler was a member of all the important academies of his time and received many awards.

Second Petersburg period

1766 Frederick II's bumbling was influential in Euler's accepting an offer from Catherine the Great to return to St. Petersburg, where he remained until his death.

1771 In the aftermath of a failed cataract operation, Euler lost the sight of his remaining good eye and soon became nearly completely blind. During the great St. Petersburg fire, he was saved from his burning house at the last moment by the Basler artisan Peter Grimm. Yet, amazingly, his productivity increased: approximately half of his prodigious output occurred during this second Petersburg period, including three-volume works on integral calculus and optics (Institutiones calculi integralis and Dioptrica) as well as the authoritative version of his work on algebra.

1773 Following the death of his wife Katherina, in 1776 Euler married her half-sister Abigail Gsell.

1783 On 18 September Euler suffered a stroke and died quickly and painlessly.

Euler's contributions to mathematics cover a wide range, including analysis and the theory of numbers. He also investigated many topics in geometry.

Rigid body Kinematics

Euler's contributions to mathematics cover a wide range, including analysis and the theory of numbers. He also investigated many topics in geometry with application to the .kinematics. Euler equations in the area of rigid body kinematics - rotation about a fixed point - are very important and applicable in future development in dynamics.

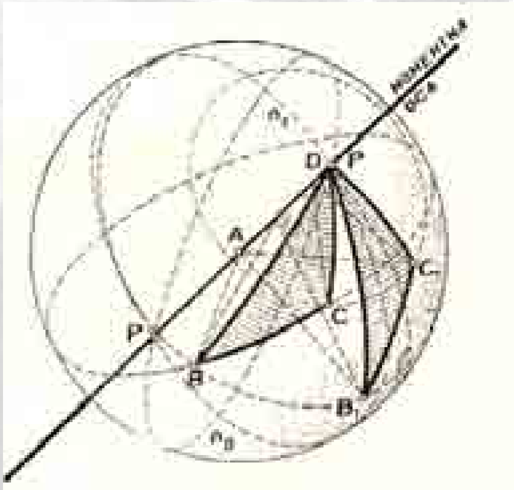


Figure 4. Sphere geometry of rotation

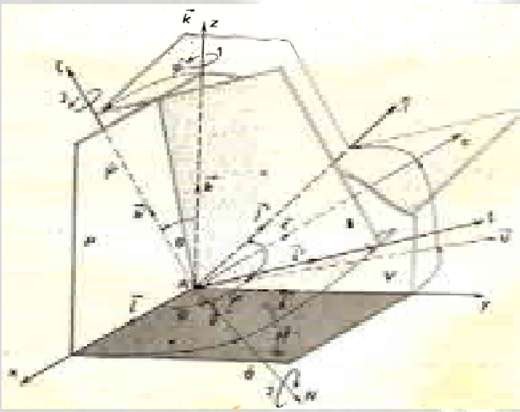


Figure 5. Kinematics of the rigid body rotation around a fixed point- Euler's angles: ψ angle of precession, ϑ angle of nutation φ angle of 8 self rotation)

New Vector to Rigid body rotation around a fixed point and instantaneous axis through a fixed point

Let us consider the special case of rotation around the fixed point O and around the moving axis oriented by the unit vector $\vec{n}$ with the rotation around the fixed point O using Euler's angles: angles ψ of precession , angle ϑ of nutation an angle φ of self rotation as well as the mass moment vectors conected by the fixed point O and the rotating axis, also around the fixed point.

Using Euler's angles (ψ,ϑ,φ), the Euler angular velocities are defined as follows: $\dot{\psi}$ - anglular velocity of precession, is defined by $\vec{\omega}_{\psi}=\dot{\psi}\vec{k}$ in the direction of $\vec{k}$, $\dot{\theta}$ - angular velocity of nutation is defined by $\vec{\omega}_{\vartheta}=\dot{\vartheta}\vec{e}$ in the direction of $\vec{e}$ (knot axis) and $\dot{\varphi}$ -angular velocity of self rotation is defined by $\vec{\omega}_{\varphi}=\dot{\varphi}\vec{k}'$ in the direction of $\vec{k}'$ (axis of body self rotation). Overall instantaneous angular velocity of the body rotation around the fixed point is:

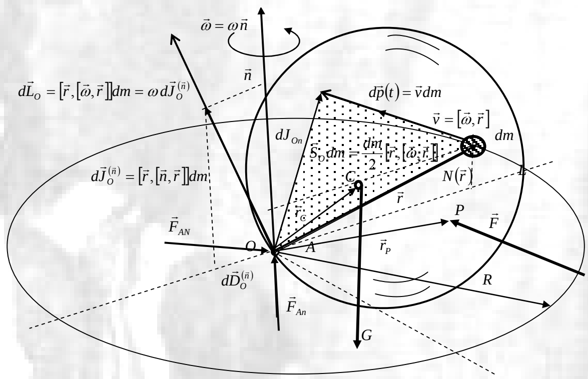
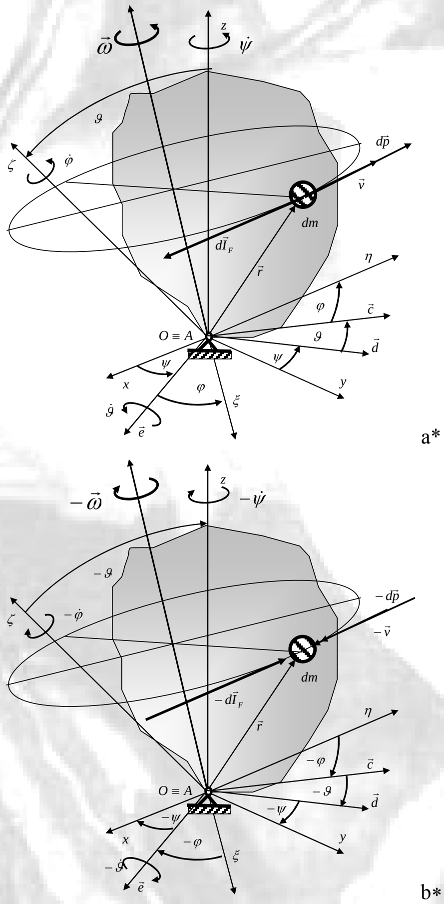


Figure 6. Rigid body dynamics around the fixed point. Mass moment vectors coupled for the fixed point and the instantaneous axis and Linear momentum and angular momentum for the fixed point and the rigid body dynamics

$$\vec{\omega}=\dot{\vartheta}\vec{e}+\dot{\psi}\vec{k}+\dot{\varphi}\vec{k}',$$

with the components in the fixed coordinate system $Oxyz$ (see Fig.7):



a*

b*

Figure 7. a∗ and b∗ Dynamics of the rigid body rotation around the fixed point- Euler's angles: ψ - roll, ϑ - pitch and φ - yaw.

(ψ angle of precession , ϑ angle of nutation an φ angle of self rotation)

$$\vec{\omega}_x = \vec{i}\left(\dot{\vartheta}\cos\psi + \dot{\varphi}\sin\psi\sin\vartheta\right),$$

$$\vec{\omega}_y = \vec{j}\left(\dot{\vartheta}\sin\psi - \dot{\varphi}\cos\psi\sin\vartheta\right),$$

$$\vec{\omega}_z = \vec{k}\left(\dot{\psi} + \dot{\varphi}\cos\vartheta\right).$$

or in the scalar form

$$\omega_x = \dot{\vartheta}\cos\psi + \dot{\varphi}\sin\vartheta\sin\psi$$

$$\omega_y = \dot{\vartheta}\cos\psi - \dot{\varphi}\sin\vartheta\cos\psi$$

$$\omega_z = \dot{\psi} + \dot{\varphi}\cos\vartheta$$

The components of the instantaneous angular velocity in the moving coordinate system $O\xi\eta\zeta$ fixed with the rigid body are (see Fig.7):

$$\vec{\omega}_\xi = \vec{i}'\left(\dot{\vartheta}\cos\varphi - \dot{\psi}\sin\varphi\sin\vartheta\right),$$

$$\vec{\omega}_\eta = \vec{j}\left(\dot{\vartheta}\sin\varphi + \dot{\psi}\cos\varphi\sin\vartheta\right),$$

$$\vec{\omega}_\zeta = \vec{k}'\left(\dot{\varphi} + \dot{\psi}\cos\vartheta\right)$$

or in the scalar form

$$\omega_\xi = \dot{\vartheta}\cos\varphi - \dot{\psi}\sin\varphi\sin\vartheta$$

$$\omega_\eta = \dot{\vartheta}\sin\varphi + \dot{\psi}\cos\varphi\sin\vartheta$$

$$\omega_\zeta = \dot{\varphi} + \dot{\psi}\cos\vartheta$$

The linear momentum (impuls of the motion) and the angular momentum (kinetic moment of the motion) of the body rotation around the fixed point O expressed by the body mass moment vectors are in the following forms:

– ∗The linear momentum (impulse of the body mass motion) $\vec{p}(t)$ of motion of a material system, or a rigid body rotating around a fixed axis $\vec{n}$ with the angular velocity $\vec{\omega} = \omega\vec{n}$ is given in the following form:

$$\vec{p}(t) = \omega\iiint_V [\vec{n},\vec{r}]\,dm = \omega[\vec{n},\vec{r}_C]M = \omega\vec{S}_0^{(\vec{n})},$$

where

$$\vec{S}_0^{(\vec{n})} = \iiint_V [\vec{n},\vec{r}]\,dm.$$

$\vec{S}_0^{(\vec{n})}$ is the mass linear moment of the body with respect to the pole O and for the axis oriented by the unit vector $\vec{n}$, passing through the point O (see Refs. Hedrih 1991, 1992, 1993a,b, 1998a,b,c, 2001,2007).

– ∗Angular momentum $\vec{L}_0$ for the rigid body rotating around the fixed axis oriented by the unit vector $\vec{n}$, through the pole O, with the angular velocity $\vec{\omega} = \omega\vec{n}$ is in the following form:

$$\vec{L}_0 = \omega\iiint_V [\vec{r},[\vec{n},\vec{r}]]\,dm = \omega\vec{J}_0^{(\vec{n})}$$

where we introduce the following notation

$$\vec{J}_O^{(\vec{n})} \overset{def}{=} \iiint_V [\vec{r},[\vec{n},\vec{r}]]\,dm$$

for the mass inertia moment vector $\vec{J}_0^{(\vec{n})}$ for the pole O and the fixed axis oriented by the unit vector $\vec{n}$.

The mass inertia moment vector $\vec{J}_0^{(\vec{n})}$ has two components: one component J_{0n} is in the rotation axis direction and corresponds to the axial body mass inertia moment and the second component $\vec{\mathbf{D}}_0^{(\vec{n})}$ is orthogonal to the rotation axis and is the deviational component in the deviational plane. $\vec{\mathbf{J}}_0^{(\vec{n})}$ can be expressed in the following form (see Refs. Hedrih 1998a,b,c, 2001,2007):

$$\vec{\mathbf{J}}_0^{(\vec{n})} = \left(\vec{n},\vec{J}_{0O}^{(\vec{n})}\right)\vec{n} + \left[\vec{n},\left[\vec{\mathbf{J}}_0^{(\vec{n})},\vec{n}\right]\right] = J_{0n}\vec{n} + \vec{\mathbf{D}}_0^{(\vec{n})}.$$

Kinetic energy for that case of the model motion is:

$$\left(\vec{\omega},\vec{L}_0\right) = \omega^2\iiint_V \left(\vec{n},[\vec{r},[\vec{n},\vec{r}]]\right)dm = \omega^2\left(\vec{n},\vec{J}_0^{(\vec{n})}\right)$$
$$= \omega^2\iiint_V [\vec{n},\vec{r}]^2\,dm = \omega^2 J_{0n}^{(\vec{n})} = 2E_k \quad .$$

For obtaining the necessary mass moment vector $\vec{\mathbf{J}}_0^{(\vec{n})}$ for the rotation axis oriented by the unit vector $\vec{n}$ through the pole O, or the mass moment vector $\vec{\mathbf{J}}_C^{(\vec{n})}$ for the parallel axis oriented by same unit vector $\vec{n}$ through the rigid body mass center C, we can use the following vector expressions:

$$\vec{\mathbf{J}}_0^{(\vec{n})} = \iiint_V [\vec{r},[\vec{n},\vec{r}]]\,dm = \vec{\mathbf{J}}_0^{(\vec{i})}\cos\alpha + \vec{\mathbf{J}}_0^{(\vec{j})}\cos\alpha + \vec{\mathbf{J}}_{0C}^{(\vec{k})}\cos\gamma$$

$$\vec{\mathbf{J}}_C^{(\vec{n})} = \iiint_V [\vec{\rho},[\vec{n},\vec{\rho}]]\,dm = \vec{\mathbf{J}}_C^{(\vec{i})}\cos\alpha + \vec{\mathbf{J}}_C^{(\vec{j})}\cos\alpha + \vec{\mathbf{J}}_{0C}^{(\vec{k})}\cos\gamma$$

$$\vec{\mathbf{J}}_0^{(\vec{n})} = \vec{\mathbf{J}}_C^{(\vec{n})} + [\vec{r}_C,[\vec{n},\vec{r}_C]]M$$

where $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are cosines of the direction of the unit vector $\vec{n}$ rotating axis orientation with respect to the corresponding system coordinates, M is the rigid body mass, and $\vec{r}_C$ is the vector position of the rigid body mass center.

For this considered case of the material system model rotation around the fixed point, it is necessary to point out that both vectors of mass moments are changeable with respect to the fixed point and instantaneous axis of rotation. This happens because the instantaneous axis and the rigid body change their relative positions and the relation to each other during the body rotation motion around the fixed point.

Euler equations in the vector form, using the mass moment vectors coupled to the fixed point and the instantaneous axis are:

$$\frac{d\vec{p}(t)}{dt} = \dot{\omega}\vec{S}_0^{(\vec{n})} + \omega\dot{\vec{S}}_0^{(\vec{n})} + \omega\left[\vec{\omega},\vec{S}_0^{(\vec{n})}\right] = \sum_{k=1}^{S}\vec{F}_k + \vec{G} + \vec{F}_{AN} + \vec{F}_{An}$$

$$\frac{d\vec{L}_0}{dt} = \dot{\omega}\vec{J}_0^{(\vec{n})} + \omega\overset{*}{\vec{J}}{}_0^{(\vec{n})} + \omega\left[\vec{\omega},\vec{J}_0^{(\vec{n})}\right] = \sum_{k=1}^{S}\left[\vec{r}_k,\vec{F}_k\right] + \left[\vec{r}_C,\vec{G}\right], \quad \text{(A)}$$

as well as

$$\frac{d\vec{L}_0}{dt} = \overset{*}{\vec{L}} + \left[\vec{\omega},\vec{L}_0\right] = \sum_{k=1}^{S}\left[\vec{r}_k,\vec{F}_k\right] + \left[\vec{r}_C,\vec{G}\right] = \vec{M}_0, \quad \text{(B)}$$

This previous vector equation (B) is equation of the motion and it is possible to express it in the scalar form by three differential equations of Euler's type:

$$\dot{L}_{0\xi} + \omega_\eta L_{0\zeta} - \omega_\zeta L_{0\eta} = \\ = \sum_{k=1}^{S}\left(\left[\vec{r}_k, \vec{F}_k\right].\vec{i}'\right) + \left(\left[\vec{r}_C, \vec{G}\right], \vec{i}'\right) = \left(\vec{M}_0, \vec{i}'\right) = M_{0\xi}$$

$$\dot{L}_{0\eta} + \omega_\zeta L_{0\xi} - \omega_\xi L_{0\zeta} = \\ = \sum_{k=1}^{S}\left(\left[\vec{r}_k, \vec{F}_k\right], \vec{j}'\right) + \left(\left[\vec{r}_C, \vec{G}\right], \vec{j}'\right) = \left(M_0, \vec{j}'\right) = M_{0\eta}$$

$$\dot{L}_{0\zeta} + \omega_\xi L_{0\eta} - \omega_\eta L_{0\xi} = \\ = \sum_{k=1}^{S}\left(\left[\vec{r}_k, \vec{F}_k\right], \vec{k}'\right) + \left(\left[\vec{r}_C, \vec{G}\right], \vec{k}'\right) = \left(\vec{M}_0, \vec{k}'\right) = M_{0\zeta}$$

or: in the following shorter form:

$$\dot{L}_{0\xi} + \omega_\eta L_{0\zeta} - \omega_\zeta L_{0\eta} = M_{0\xi} ,$$

$$\dot{L}_{0\eta} + \omega_\zeta L_{0\xi} - \omega_\xi L_{0\zeta} = M_{0\eta} ,$$

$$\dot{L}_{0\zeta} + \omega_\xi L_{0\eta} - \omega_\eta L_{0\xi} = M_{0\zeta} .$$

For the case in which the coordinate axes of the moving coordinate system fixed to the rigid body are body principal mass inertia moment axes, from the previous Euler system of differential equations we have:

$$\dot{\omega}_1 J_{01} - \omega_2\omega_3\left(J_{02} - J_{03}\right) = M_{01} ,$$

$$\dot{\omega}_2 J_{02} - \omega_1\omega_3\left(J_{02} - J_{01}\right) = M_{02} ,$$

$$\dot{\omega}_3 J_{03} - \omega_1\omega_2\left(J_{01} - J_{02}\right) = M_{03} .$$

The first vector equation from the system (A) of the vector equations is possible to obtain kinetic pressure to the fixed point (bearing, or to kinetic reaction or necessary action by the body).

Citations

* The most cited mathematician of all times
* Over 20 formulae of elementary mathematics bear Euler's name
* An even greater number of formulas and notions of advanced mathematics bears his name.
* The most fruitful mathematician of all times (800 pages per year)

Some elementary Euler formulae and theorems

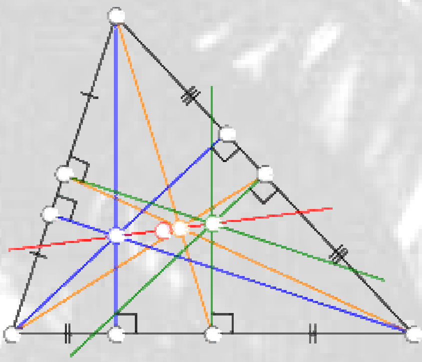


Figure 16. Euler's line

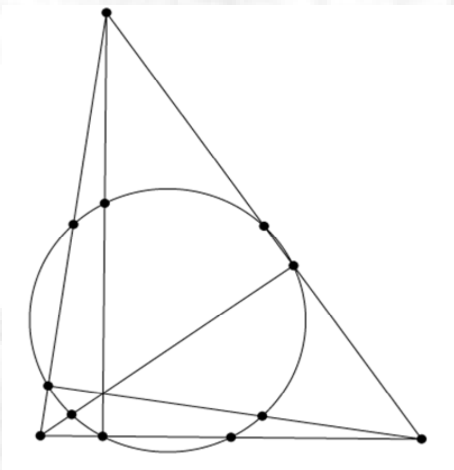
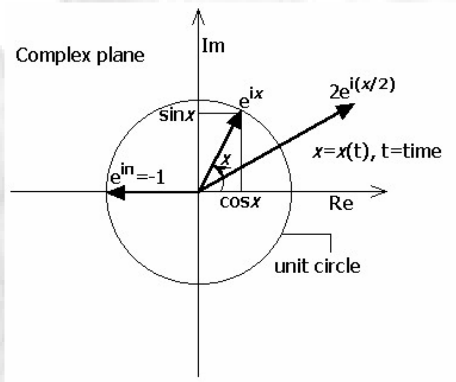


Figure 8. Euler's line



$$e^{ix} = \cos x + i\sin x$$

Figure 9. Euler's formula

Graph theory

Figure 10. The problem of seven Koenigsberg bridges

If all nodes are even, the graph can be drawn by starting from each of them (1736).

A graph with more than two odd nodes cannot be drawn in a single stroke.

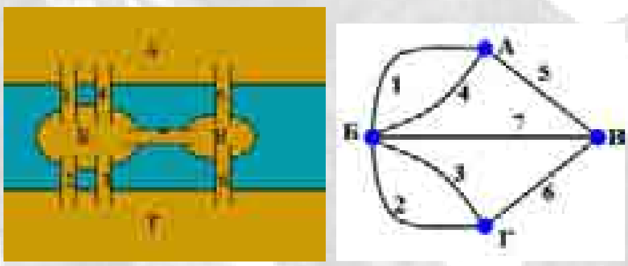


Figure 11. Examples of graphs.

Concluding Remarks

Euler's program for mechanics presented in the treatise (Mechanics or the analytical representation of the science of motion) paved the way for a successful development of mechanics in the 18th century. In contrast to Newton's geometry-related procedure in the Principia, Euler formulated mechanical laws preferentially in terms of the differential calculus. Euler claimed that "those laws of motion which a body observes when left to itself in continuing either rest or motion pertain properly to infinitely small bodies". Geometrically, these bodies can be considered as points, but mechanically they are less than any extended body, but different from mathematical points due to their finite mass. Analytically, motion is described in terms of infinitesimal time intervals whereas, geometrically, it is related to straight lines and planes as basic elements.

In fluid dynamics, the **Euler equations** govern the compressible, in viscid flow. They correspond to the Navier-Stokes equations with zero viscosity and heat conduction terms. They are usually written in the conservation form shown below to emphasize that they directly represent conservation of mass, momentum, and energy.

Momentum Equation for Frictionless Flow: Euler's Equation

Euler's Equation

$$\rho \frac{D\vec{V}}{Dt} = \rho \vec{g} - \nabla p$$

In the Cartesian coordinate system

$$\rho = \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = \rho g_x - \frac{\partial p}{\partial x}$$

$$\rho = \left(\frac{\partial \upsilon}{\partial t} + u \frac{\partial \upsilon}{\partial x} + \upsilon \frac{\partial \upsilon}{\partial y} + w \frac{\partial \upsilon}{\partial z} \right) = \rho g_z - \frac{\partial p}{\partial z}$$

$$\rho = \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + \upsilon \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = \rho g_z - \frac{\partial p}{\partial z}$$

In the cylindrical coordinate system

$$\rho a_r = \rho \left(\frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} + \frac{V_\theta}{r} \frac{\partial V_r}{\partial \theta} + V_z \frac{\partial V_r}{\partial z} - \frac{V_\theta^2}{r} \right) = \rho g_r - \frac{\partial p}{\partial r}$$

$$\rho a_\theta = \rho \left(\frac{\partial V\theta}{\partial t} + V_r \frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r} \frac{\partial V_\theta}{\partial \theta} + V_z \frac{\partial V_\theta}{\partial z} - \frac{V_r V_\theta}{r} \right)$$

$$= \rho g_\theta - \frac{1}{r} \frac{\partial p}{\partial \theta}$$

$$\rho a_z = \rho \left(\frac{\partial V_z}{\partial t} + V_r \frac{\partial V_z}{\partial r} + \frac{V_\theta}{r} \frac{\partial V_z}{\partial \theta} + V_z \frac{\partial V_z}{\partial z} \right) = \rho g_z - \frac{\partial p}{\partial z}$$

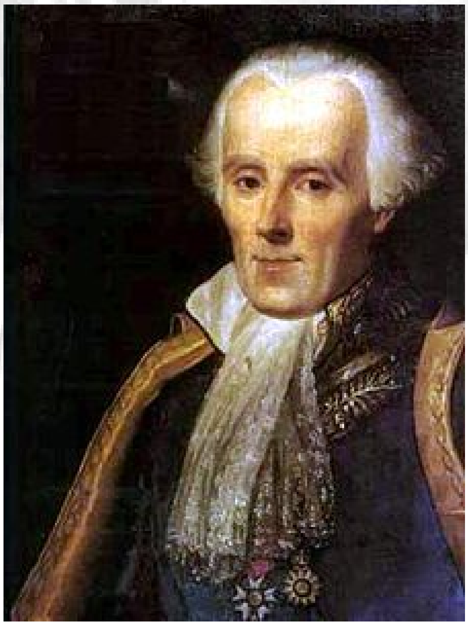
Continuity

$$\nabla \cdot \vec{V} = 0$$

Euler's contributions to Rational or Mathematical Fluid Mechanics are important contributions as well as his contribution to the General Theory of the Motion of Fluids. Euler's generalization of the stream function concept to a pair of stream functions or stream surfaces is also the result of his research. Euler's Potential had been formulated about 100 years before a similar detailed mathematical exposure was formulated by Jacobi and Clebsch around 1844. Furthermore, Euler's work initiated the establishment of naval science in Russia and influenced the art of building naval ships in Russia in the 18th century in particular. An overview about Euler's accomplishments in Naval Architecture and Ship Hydrodynamics at Russia's St. Petersburg Academy of Sciences is also an important contribution to world science and to mechanical engineering.

The theory of magnetism for which he was awarded the Paris Academy Prize in 1744 is also very well known. Euler began with Descartes' idea that all magnetic phenomena are elicited from the circulation of imperceptible conduits throughout the corpuscular magnetic body. Euler imagined that the magnetic body possessed pores which formed continuous piping, parallel and bristling, similar to veins or valves and so narrow as to only allow passage for the most subtle parts of the ether, the elasticity of which pushes the relaxed parts into the magnet pores. Then the force causes it to bend onto itself at the exit only to return again and form a type of vortex. Through this ingenious idea which was developed after much thought, Euler was able to explain magnetic phenomenon. The hypothesis was proved out by experiments and these conformed to natural laws which in turn ensured its ultimate probability.

The Opera Omnia



"Read Euler, read Euler, he is the teacher (master) of us all".
Pierre-Simon, marquis de Laplace (March 23, 1749 - March 5, 1827)

Published by Birkhauser and the **Euler Commission** of Switzerland, the **Opera Omnia** is the definitive printed source for Euler's works. The publication began in 1911, and to date 76 volumes have been published, comprising almost all of Euler's works.

Principle:

"*Mathematicians do not have biography, they have bibliography*"

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The URL of this page is:
http://www-history.mcs.st-andrews.ac.uk/Biographies/Euler.html
http://www.leonhard-euler.ch/
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Leonardo Ojler (1707-1783) i mehanika krutog tela

Uvodni tekst ovog broja časopisa posvećen 300. godišnjici od rođenja Leonarda Ojlera, jednog od najpoznatijih naučnika XVIII veka iz oblasti matematike i mehanike. Rođen je 15. aprila 1707 godine u Bazelu, Švajcarska. Zahvaljujući znamenitom Bernuliju, koji je uticao na njegovog oca da promeni svoju odluku, posvetio se matematici umesto teologiji. Iako je veći deo njegovog rada bio iz oblasti elementarne matematike dao je značajan naučni doprinos i u oblasti astronomije i fizike.
U svojim radnim dvadesetim godinama Ojler je izgubio oko. Kasnije je ostao i bez drugog oka, ali je uprkos tome bio vrlo produktivan. Tokom života objavio je oko 500 knjiga i radova, a još oko 400 radova objavljeno je posthumno. Laplas je govorio svojim studentima: "Čitajte Ojlera, čitajte Ojlera! on je gospodar sviju nas"

Leonhard Euler (1707-1783) et la mécanique du corps solide

L'éditorial de ce numéro est dédié au 300^{ième} anniversaire de la naissance de Leonhard Euler, savant célèbre du 18^{ième} siècle, très connu pour ces travaux en mathématique et mécanique. Il est né le 15 avril 1707 à Bâle, en Suisse. Il devait, selon le désir de son père, étudier la théologie, mais grâce à célèbre savant Bernoulli qui a influencé le père de Leonhard, celui-ci a pu se consacrer aux mathématiques. Bien que la majeure partie de son travail concerne les mathématiques élémentaires, sa contribution est considérable en astronomie et en physique. A l'âge de vingt ans Euler a perdu un œil; plus tard il a perdu son autre œil, mais malgré cela il était très productif. Au cours de sa vie il a publié environ 500 livres et travaux; après sa mort, on a publié encore 400 travaux. Laplace disait souvent à ses étudiants: "Lisez Euler, lisez Euler! Il est maître de nous tous".

Leonhard Euler