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Nonlinear Dynamics and Aleksandr Mikhailovich Lyapunov

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Paper is dedicated to 150th anniversary of academician A.M. Lyapunov. His work concentrated on the stability of equilibrium, motion of the mechanical system and the stability of a uniformly rotating fluid. He devised important methods of approximation. Lyapunov's methods introduced in 1899 provide ways of determining the stability of sets of ordinary differential equations. Nonlinear dynamics as a mode to address dynamical systems is stressed. Basic terms of Nonlinear Dynamics sourcing from Lyapuniv's scientific results and named after him are presented.

Introduction

NONLINEAR dynamics is a mode to address dynamical systems.

Basic terms of nonlinear dynamics are: dynamical system, state phase space, dynamics or equation of motion, orbit or trajectory, flow, Poincaré section and Poincaré map, non-wandering set.

A set of points in the ***phase space*** have the following property: All ***orbits*** starting from a point of this set come arbitrarily close and arbitrarily often to any point of the set. Non-wandering sets come in four variations: fixed points, limit cycles, quasi periodic orbits and chaotic orbits. The first three types can also occur in linear dynamics. The fourth type appears only in nonlinear systems, whose possibilities were first anticipated by the genius of Henri Poincaré (1854-1912). In the seventies, this irregular behaviour was termed ***deterministic chaos***. In the ***Poincaré*** map, limit cycles become fixed points. A non-wandering set can be either stable or unstable. Changing a parameter of the system can change the stability of a non-wandering set. This is accompanied by a change of the number of non-wandering sets due to the bifurcation.

Lyapunov exponents, Lyapunov spectrum and Lyapunov function

While playing with the ***driven pendulum,*** for e.g., irregular sequences of left and right turns may be noticed. This behavior, called ***deterministic chaos***, is ***the most prominent effect of nonlinear dynamics***. The term "*deterministic chaos*" seems to be ***a contradiction in itself***. *How can something be deterministic and chaotic at the same time?*

The key element of deterministic chaos is *the sensitive dependence of the* ***trajectory*** *on the initial conditions.* As an example, the plot on the first 15 seconds of a *demonstration* of this effect for a horizontally driven pendulum with two different initial conditions is characteristics. Both initial conditions differ in one arcsec. only. Nevertheless, 10 seconds later they behave totally different. In the beginning, the distance of the trajectories increases on average exponentially (it is visible by the distance in phase space). The rate of divergence is measured by the *largest* ***Lyapunov exponent***.

A ***quantitative measure*** of the ***sensitive dependence on the initial conditions*** is the ***Lyapunov exponent***. It is the averaged rate of ***divergence*** (or ***convergence***) of ***two neighboring trajectories***. Actually, there is a whole spectrum of Lyapunov exponents. Their *number* is equal to the ***dimension of the phase space***. When speaking about *the* Lyapunov exponent, ***the largest*** one is meant. It is important because it determines the ***prediction horizon***. Even qualitative predictions are impossible for a time interval beyond this horizon.

The ***maximal Lyapunov exponent*** can be defined as follows:

$$\lambda = \lim_{t\to\infty} \frac{1}{t} \ln \frac{|\delta Z(t)|}{\delta Z_0}$$

The Lyapunov spectrum. For a dynamical system with evolution equation F^T in a n–dimensional phase space, the spectrum of Lyapunov exponents

$$\{\lambda_1, \lambda_2, ..., \lambda_n\},$$

in general, depends on the starting point x_0. The Lyapunov exponents describe the behavior of vectors in the tangent space of the phase space and are defined from the Jacobian matrix

$$J^t(x_0) = \left.\frac{df^t(x)}{dx}\right|_{x0}$$

If the system is ***conservative***, a volume element of the phase space will stay the same along a trajectory. Thus ***the sum of all Lyapunov exponents must be zero.*** If the system is ***dissipative***, the sum of Lyapunov exponents is ***negative.*** If the system is ***a flow***, one exponent is always zero-the Lyapunov exponent corresponding to ***the eigenvalue*** of L with an eigenvector in the direction of the flow.

The Lyapunov spectrum can be used to give an estimate of the rate of entropy production and of the ***fractal dimension*** of the considered ***dynamical system***. In particular, from the knowledge of the Lyapunov spectrum it is possible to obtain the so-called Kaplan-Yorke dimension D_{KY}, D_{KY} represents an upper bound for the information dimension of the system. Moreover, the sum of all the positive Lyapunov exponents gives an estimate of the Kolmogorov-Sinai entropy accordingly to Pesin's theorem. The inverse of the largest Lyapunov exponent is sometimes

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referred to as Lyapunov time in references and defines the characteristic e-folding time. For chaotic orbits, the Lyapunov time will be finite, whereas for regular orbits it will be infinite.

Whereas the (global) ***Lyapunov exponent*** gives a measure for the total predictability of a system, it is sometimes interesting to estimate the local predictability around a point x_0 in the phase space. This may be done through the ***eigenvalues*** of the ***Jacobian*** matrix $J^{\circ}(x_0)$. These eigenvalues are also called local Lyapunov exponents. ***The eigenvectors*** of the Jacobian matrix point in the direction of the ***stable*** and ***unstable*** manifolds.

In mathematics, ***Lyapunov fractals*** (also known as *Markus-Lyapunov fractals*) are bifurcational fractals derived from an extension of the ***logistic map*** in which the degree of growth of the population, *r*, periodically switches between two values *a* and *b*. A ***Lyapunov fractal*** is constructed ***by mapping*** the ***regions*** of ***stability and chaotic behavior*** (measured using the Lyapunov exponent λ) in the *a*-*b* plane for a given ***periodic sequence*** of *a*s and *b*s.

In mathematics, especially in *stability theory*, **Lyapunov functions**, named after *Aleksandr Mikhailovich Lyapunov*, are functions which can be used to prove **the stability or instability** of the fixed points in *dynamical systems* and *autonomous differential equations*.

There is no general method to construct a Lyapunov function and the inability to find a Lyapunov function is inconclusive with respect to stability or instability. For ***physical systems***, ***conservation laws*** can often be used **to construct** a Lyapunov function.

The fixed point p is ***stable*** if there exists a Lyapunov function on an open neighbourhood U of p. The point is ***asymptotically stable*** if there exists a strong Lyapunov function on an open neighbourhood U of p.

Aleksandr Mikhailovich Lyapunov (1857 – 1918) - scientific work

Aleksandr Mikhailovich Lyapunov (Александр Михайлович Ляпунов) was born June 6, 1857 in Yaroslavm, Russia and died November 3, 1918 in Odessa, Russia. He was a Russian mathematician, mechanician and physicist. Sometimes his name is also written as Ljapunov, Liapunov or Ljapunow, and often improperly pronounced "La-yapunov".

He studied at the Physico-Mathematical department of the University of Saint Petersburg, where he was a schoolfellow of Markov. In the beginning, he listened to Mendeleyev's lectures on chemistry. After a month he transferred to the mathematics department of the University, but he continued attending the chemistry lectures. Mathematics was taught at that time by Chebyshev and his students Aleksandr Nikolaevich Korkin and Yegor Ivanovich Zolotarev. In his fourth year he received the gold medal for a work on hydrostatics, which had been suggested by the faculty. As was said by Vladimir Andreevich Steklov, "*Chebyshev saw in the young man such an immense research power, that he had dared to lay on him such a toilsome task*".

Lyapunov wrote his first independent scientific works under the guidance of professor of mechanics, D. K. Bobylev. His first work on hydrostatics was the basis for his first published scientific paper entitled: "*About the equilibrium of solid bodies in vessels with arbitrary forms, filled with dense fluids (О равновесии тяжелых тел в тяжелых жидкостях, содержащихся в сосуде определенной формы*) and "*About the potential of hydrostatic pressure" (О потенциале гидростатических давлений*). In both papers, he developed new rigorous proofs of a few earlier incomplete hydrostatics theorems with many new approaches and original results.

His first scientific papers with original results gained him the title of a ***candidate in mathematical sciences.***

He graduated in 1880 and received a Master's degree in *applied mathematics* in 1884 with the thesis entitled: "*About the stability of elliptic forms in the equilibrium of turbulent fluid*" *(Об устойчивости эллипсоидальных форм равновесия вращающейся жидкости*). This thesis treated an important and difficult research task - the shapes of celestial bodies. This research task was offered by Chebyshev to Zolotarev and ***Sofia Vasilyevna Kovalevskaya*** and ***Chebyshev*** was aware of the difficulty.

Lyapunov had already begun to study in what was to become his main research direction - stability in his previous two-year long attempt to solve the task. After the public announcement, his original results in stability instantly attracted the attention of mathematicians, mechanicians, physicists and astronomers all over the world.

In 1885 he became a private reader of the Kharkov University in the department of mechanics.

From 1899 to 1902 he was head of Kharkov mathematical society and an editor of its publication *News*. While at Kharkov University, he played a major role in the Kharkov Mathematical Society, being its vice-president from 1891 to 1898 and president from 1899 until he left Kharkov in 1902. He also edited the *Communications of the Kharkov Mathematical Society.*

Aleksandr Mikhailovich Lyapunov remained in Kharkov for 17 years. During this time, he played a leading role in running the Society and presented 27 reports on the monthly meetings. *Due to Lyapunov, mathematical research and reports at Kharkov Mathematical Society meetings reached a much deeper level.*

On December 2, 1900 he was elected a corresponding member of the Russian Academy of Sciences, and on October 6, 1901 a fully entitled member of the Academy in the field of applied mathematics.

Lyapunov had already been lecturing at the Department of Mechanics at Kharkov University from 1880 and this was taking a lot of his time. His student and collaborator, academician ***Steklov***, said about his fine lectures: "*A handsome young man, by his appearance almost like the other students, came before the audience, where there was also an old dean, professor Levakovsky, who was respected by all students. After the dean had left, the young man, his voice trembling, started to lecture about the dynamics of a point, instead of a theme from the dynamics of systems. This* *subject was already addressed in the lectures of professor Delaryu. I was in the fourth class. I had listened to the lectures in Moscow of Davidov, Cinger, Soletov and Orlov. I had been in the University of Kharkov for two years already, so I was familiar with the lectures on mechanics. But I had not known the subject from the beginning and I had never seen it in any textbook. That way, I could not have been bored with the lecture. Aleksandr Mikhailovich had earned the respect of the audience in an hour by shear power of a natural gift so seldom seen in such a young man. He was not aware of this of course. From that day on, students looked at him with different eyes and showed him special respect. Many a time they dared not speak to him, for fear of showing their own ignorance*".

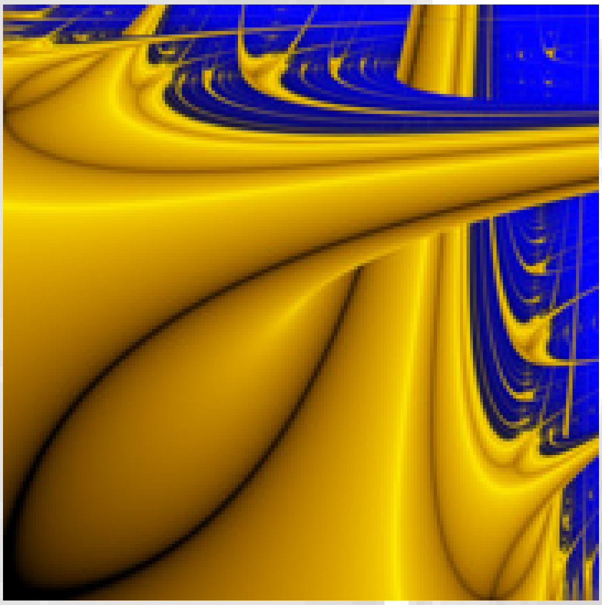

Figure 1. Lyapunov fractal From Wikipedia, the free encyclopedia

Lyapunov lectured at the University on themes from theoretical mechanics, integrals of differential equations and the theory of probability. These lectures were never published and they remained only in the notes of the students. He lectured about mechanics in six areas: kinematics, the dynamics of a pointed body, the dynamics of systems of pointed bodies, the theory of attracting forces, the theory of the deformation of solid bodies and hydrostatics. At the same time, he lectured on analytical mechanics between 1887 and 1893 at the Technological Institute at Kharkov.

In 1892 he was awarded his doctorate in science for the thesis entitled: "*A general task about the stability of motion*" (*Общая задача об устойчивости движения*). After the doctorate, Lyapunov became a full professor at Kharkov University. After the death of Chebyshev in 1894, Lyapunov became Head of applied mathematics at the University at Saint Petersburg, where he was entirely devoted to tutoring and research work.

His work from 1898: "About some questions, connected with *Dirichlet*'s tasks" (О некоторых вопросах, связанных с задачей Дирихле) contains a study of the properties of potential around charges and dipoles, continuously distributed along any surface. His work in this field is in close connection with the work of Steklov. Lyapunov developed many important approximative methods. His methods, nowadays called Lyapunov methods, developed in 1899, make it possible to define the stability of sets of ordinary differential equations. He elaborated the modern rigorous theory of the stability of a system and the motion of a mechanical system on the basis of a finite number of parameters. In probability theory, he generalized the works of Chebyshev and Markov, and he finally proved the Central limit theorem using more common conditions than his forerunners. The method he used as proof is one of the foundations of probability theory.

Lyapunov's paper entitled: "Sur le probleme general de la stabilite du mouvement" (1892) (in French) marks the beginning of stability theory.

His research results in the field of differential equations, potential theory, the stability of systems and probability theory is very important. His main preoccupations were the stability of equilibriums and the motion of mechanical systems, the model theory for the stability of uniform turbulent liquid and particles under the influence of gravity. His results in the field of mathematical physics proved to be very important for subsequent developments of this field.

Among others he wrote the following works:

"*About constant spiral motion of a rigid body in a fluid*" *(О постоянных винтовых движениях твердого тела в жидкости)* in 1890, and many articles, which were published by the Russian Academy of Sciences:

About a series in the theory of linear differential equations (Sur une série dans la théorie des équations differentielles linéaires etc.) *1902*,

Researches in the theory of celestial bodies (Recherches dans la théorie des corps célestes) *1903*,

About Clairaut's equation, etc. (Sur l'équation de Clairaut etc.) *1904*,

A new form of the theorem on the limit of probability (Nouvelle forme du théorème sur la limite de probabilité),

About a proposition in the probability theory (Sur une proposition de la théorie des probabilités) *1906*.

With his researches on celestial mechanics, Lyapunov opened a new page in the history of global science and showed the inaccuracy in the works of several foreign scientists. In particular in two papers published in 1900 and 1901, he proved ***the central limit theorem*** using a technique based on characteristic functions. Another contribution which should be mentioned is that he worked as editor for two volumes of ***Euler's collected works***. In 1908 he participated at the 4th Mathematical congress in Rome. At this time he took part in the publication of Euler's selected works and was an editor of the 18th and 19th part of this miscellany.

Lyapunov was honoured for his outstanding contributions by election to various academies such as the Accademia dei Lincei (1909) and the French Academy of Sciences (1916) an external member of the Academy in Rome and a corresponding member of the Academy of Sciences in Paris. He was also awarded honorary membership of the Universities of St Petersburg, Kharkov and Kazan. Various tributes were paid to him on the centenary of his birth. For example on 6 June 1957 Sobolev gave the lecture *On the works of A. M. Lyapunov on potential theory* in Moscow to a joint session of the Presidium of the Academy of Sciences, the divisions of technical and physical sciences of the Academy of Sciences, at the Moscow University, the Moscow Mathematical Society, the Institute of Mechanics of the Academy of Sciences and the Institute of Automatics and Telemechanics of the Academy of Sciences. The text of this lecture is given in [14].

In [14] Pavlovskaya looks at Lyapunov's work on the problem first posed by Chebyshev which was quoted above, concerning the existence of figures of equilibrium, in addition to ellipsoidal ones, of a rotating fluid under sufficiently small variations of angular velocity of revolution, which was first solved by Lyapunov in the first approximation. He later dealt with the problem of stability of fluid ellipsoids basing his investigations on the Thomson-Tait variational principle. He showed that a sufficient condition for stability is that the second and higher variations of the potential energy are positive. Lyapunov admitted that the imposition of certain additional constraints on the first variation reduced the generality of his method, but wrote:

"*But in this respect hardly any other method of investigation could be said to be completely satisfactory*".

Lyapunov established that with variation in the angular velocity of revolution Maclaurin ellipsoids pass into Jacobi ellipsoids. The transition point is an ellipsoid of bifurcation corresponding in this case to a Jacobi ellipsoid of revolution.

Chebyshev posed a question to Lyapunov which would set the agenda for one of his main lines of research over many years:

"*It is known that at a certain angular velocity ellipsoidal forms cease to be the forms of equilibrium of a rotating liquid. In this case, do they not shift into some new forms of equilibrium which differ little from ellipsoids for small increases in the angular velocity*"

In St. Petersburg, Lyapunov devoted himself completely to scientific work. He usually worked until four or five in the morning and many times even throughout the night. He returned to the problem that Chebyshev had placed before him and, in an extensive series of papers which continued until his death, developed the theory of figures of equilibrium of rotating heavy liquids.

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Nelinearna dinamika i Aleksandar Mihajlović Ljapunov

Uvodna beseda casopisa je posvećena 150-godišnjici od rodjenja akademika A.M. Ljapunova. Njegov rad je bio usmeren kako na stabilnost ravnoteže i kretanja mehaničkih sistema, tako i na stabilnost formi jednolikog obrtanja fluida. Formulisao je više značajnih metoda aproksimacija. Ljapunovljeve metode uvedene 1899. godine ukazale su put za odredjivanje stabilnosti slupova rešenja običnih diferencijalnih jednačina. Nelinearna dinamika i njeni fenomeni su osnova za izučavanje ponašanja dinamičkih sistema. Neki osnovni pojmovi nelinearne dinamike iznikli iz Ljapunovljevih naučnih rezuktata, kasnije nazvani njegovim imenom, su ovde prikazani.

Нелинейная динамика и Александр Михаилович Ляпунов

Вводная часть настоящей работы посвящена 150-летию рождения академика А.М.Ляпунова. Его работа и исследования были направлены как к устойчивости равновесия и движения механических систем, так и к устойчивости формы единообразного вращения потока. Здесь определено много значительных методов аппроксимации. Методы Ляпунова, введены в 1899-ом году, указали путь для определения устойчивости набора решений обыкновенных дифференциальных уравнений. Нелинейная динамика и её феномены представляют основы и способы для исследования напряжённых состояний динамических систем. Здесь представлены и некоторые из основных понятий нелинейной динамики, являющиеся источниками научных результатов Ляпунова, позже получившие и названия по его имени.

La dynamique non-linéaire et Aleksandr Mikhailovitch Lyapunov

Ce papier est dédié au 150ème anniversaire de l'académicien A.M.Lyapunov. Il a concentré son travail sur la stabilité d'équilibre et le mouvement du système mécanique ainsi que sur la stabilité d'uniformité du fluide tournant. Il a conçu très importantes méthodes d'approximation.Les méthodes de Lyapunov, introduites par lui-même en 1899, permettent de déterminer la stabilité des ensembles chez les équations différentielles simples. Quand on parle des systèmes dynamiques il faut souligner la dynamique non-linéaire. Les termes basiques de la dynamique non-linéaire qui sont aussi créés par Lyapunov et qui portent son nom sont présentés dans ce travail.