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*Katica (Stevanović) Hedrih*

autor naučnog rada

“Transversal Vibration of a Parametrically Excited Beam:  
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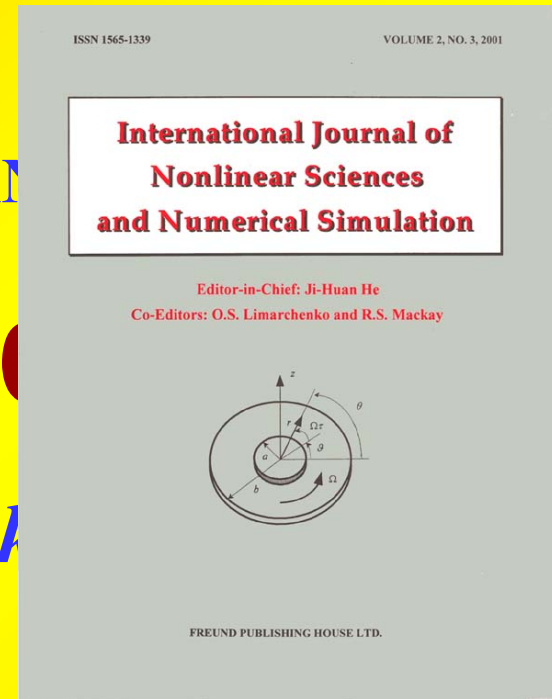
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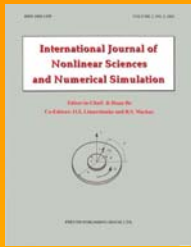
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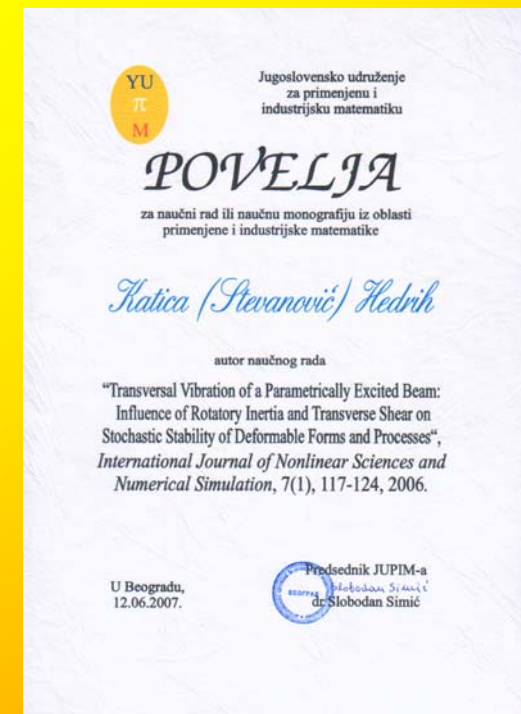
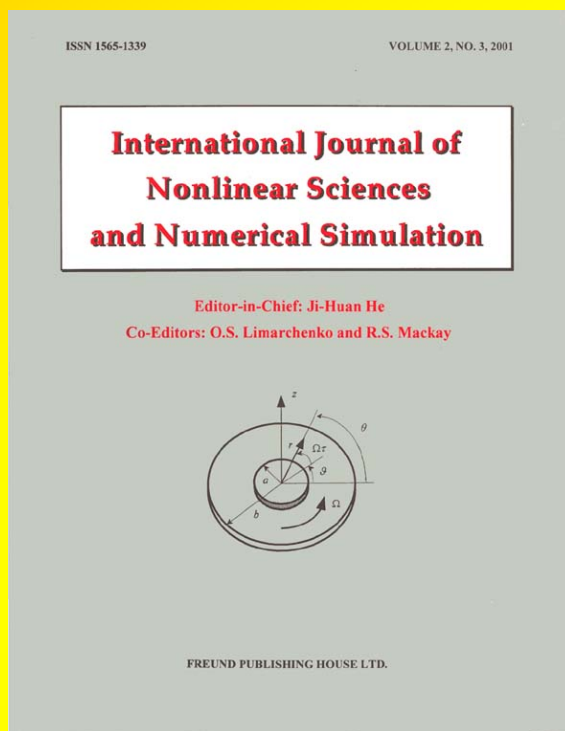
## *Influence of Rotatory Inertia and Transverse Shear on Stochastic Stability of Deformable Forms and Processes*

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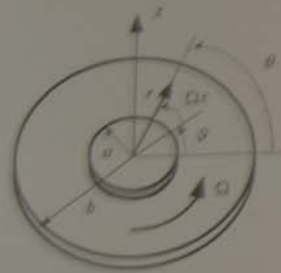
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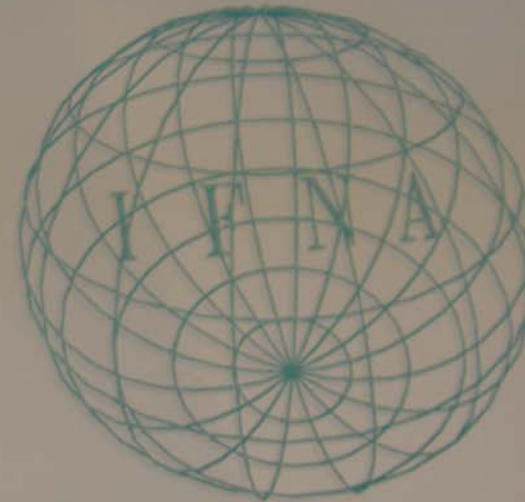
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## Transversal Vibration of a Parametrically Excited Beam: Influence of Rotatory Inertia and Transverse Shear on Stochastic Stability of Deformable Forms and Processes

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### Abstract

The partial differential equation of transversal stochastic vibration of a parametrically excited beam was derived. The beam is graded by an ideal elastic material, and it is subject to axial stochastic external excitation. The influence of rotatory inertia of beam cross section and transverse shear of beam cross section under the transverse force, and the corresponding members in the partial differential equation are taken into account. Bernoulli particular integral method and Lagrange method of variation constant are used for the transformation problem. The asymptotic averaged method is used for obtaining the first approximation of Itô stochastic differential equations. The sets of Lyapunov exponents are obtained.

**Keywords:** stochastic Itô differential equations, Lyapunov exponents, multifrequency

### 1 Introduction

The transversal vibration beam problem is classical, but in current university books on vibration, we can find only the Euler-Bernoulli's classical partial differential equation for describing transversal beam vibrations. In monograph [11] we can find a nonlinear partial differential equation for describing transversal vibrations of the beam with nonlinear constitutive stress-strain relation. By using the asymptotic method of Krilov-Bogolyubov-Mitropolskiy [10, 11], many authors studied one frequency or multi-frequency nonlinear oscillation regimes of deformable bodies. Specially, Hedrih [2, 3, 4, 5] studied one-single and two-frequency stationary and nonstationary regimes of nonlinear transversal and forced vibration of beams. Transversal vibration of the beam on the elastic Winkler's foundation under the action of multi-frequency forces with frequencies in the form of the first frequency resonant range of the beam was also studied by Hedrih[3], and some results of transversal vibration of beams graded by a creep and hereditary material were obtained in [6, 7].

In the university book [12] by Rašković, an

extended partial differential equation of transversal ideally elastic beam vibrations was presented considering the inertia rotation of the beam's cross sections and transverse shear of the cross section. Also, in numerous papers, by using the partial differential equation of the transversal ideally elastic beam vibrations with members, by which influences of the inertia rotation of the beam's cross sections and transverse shear of the cross section by transversal forces are taken into account, and based on the monograph [9] by Nowatski as the scientific source, the complex properties of the transversal vibrations of the beam are investigated.

In paper [1] stochastic stability of viscoelastic systems under bounded noise excitation was investigated. For small damping and weak random fluctuation, asymptotic expressions are derived for the Lyapunov exponent and the rotation number using the method of stochastic averaging. From the sign of the Lyapunov exponent, the condition for asymptotic stability with probability 1 of the trivial equilibrium state is obtained.

In the present paper, the stability of a pure elastic beam subjected to parametric random bounded excitations described by stochastic

$s=1,2,3,4,\dots, k=1,2$ , in the forms of expressions (35) with probability 1 for evaluation of the stability or instability, we must find the Lyapunov exponent with maximal values between Lyapunov exponents from defined sets, and determine kinetic parameters of the beam vibration such that this Lyapunov exponent is with negative values. This is not simple, because we need investigation of the  $\max \lambda'_s < 0, s=1,2,3,4,\dots, k=1,2$ . Also, we can consider the case when only one of the  $\Delta_{(s,k)} = \omega_{(s,k)} - \frac{\Omega}{2}, s=1,2,3,4,\dots, k=1,2$  is equal to zero, and all other different from zero; this analysis needs a large discussion.

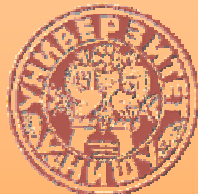
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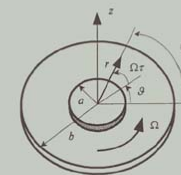


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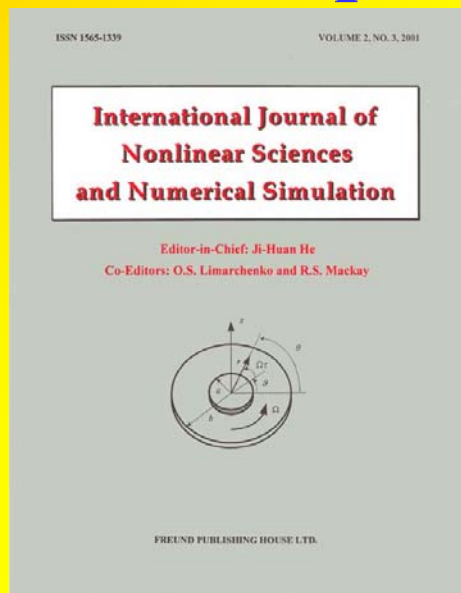
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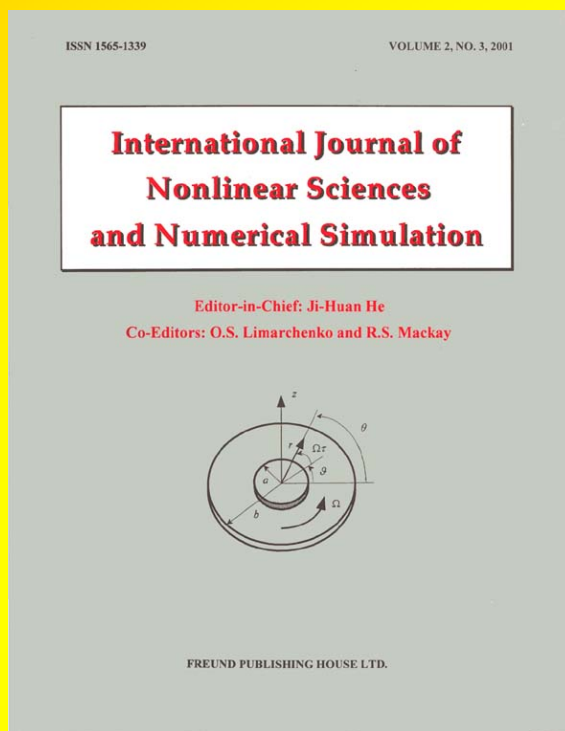
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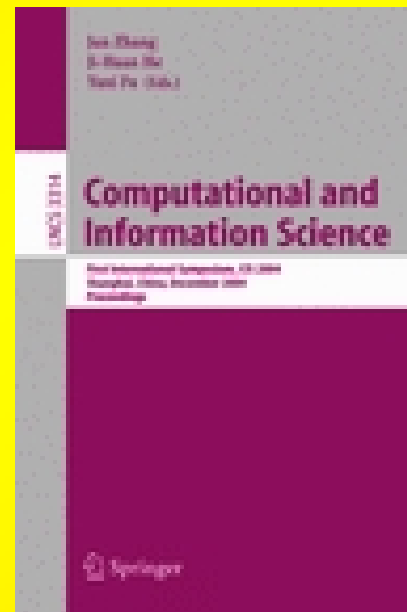
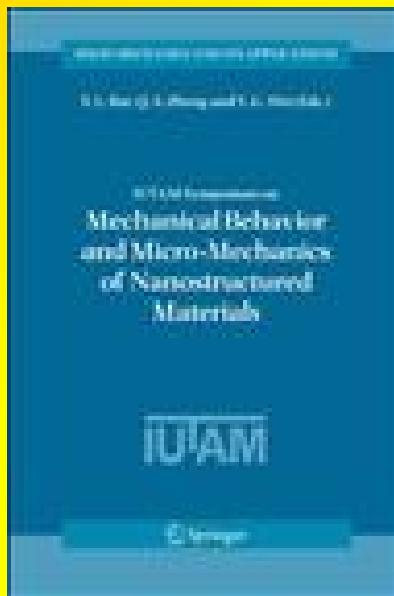
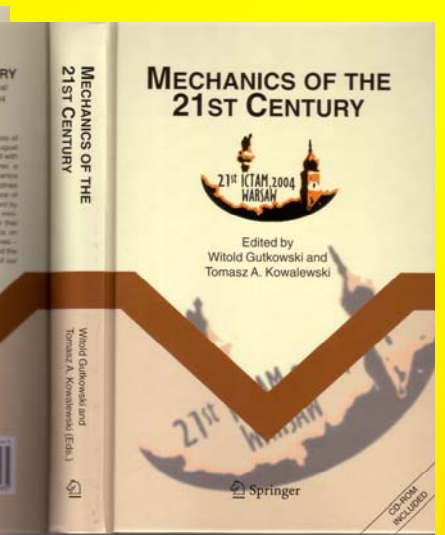
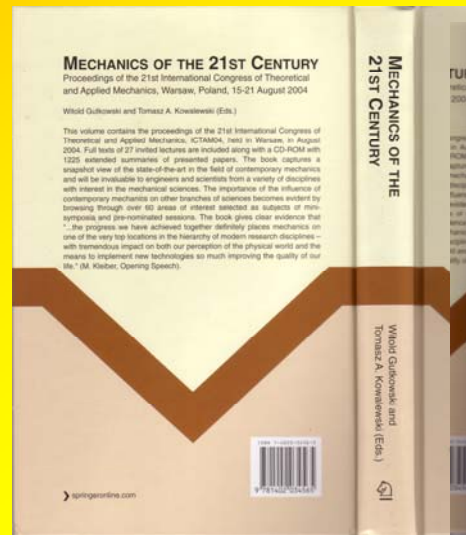
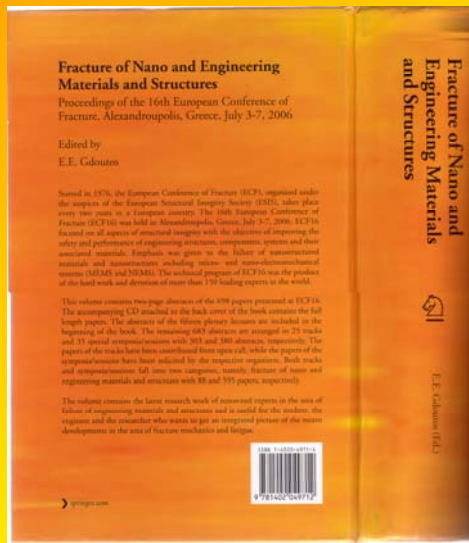
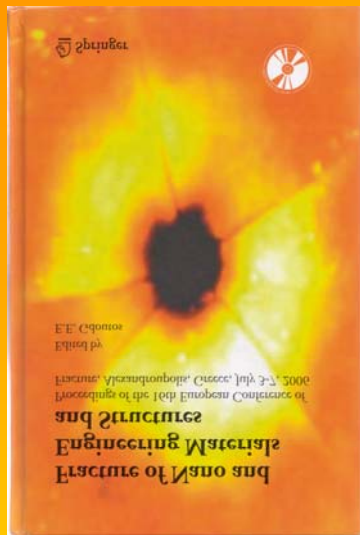




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Fractional differentiation, also called non-integer differentiation, is a concept that dates back to the beginning of differential calculus when it came to the attention of Leibniz and L'Hospital (1695) who exchanged letters about the half-order derivative. Since then many famous mathematicians and physicists have studied fractional integrals and derivatives mainly from a theoretical point of view, the main ideas being related to the names of Abel, Liouville, Riemann, Grunwald and Letnikov. By the beginning of the twentieth century only a few applications had been proposed, namely by O. Heaviside (1880) and A. Gemant (1936) who developed fractional models for electrical and mechanical engineering applications. In spite of these contributions this area of research remained almost unknown for many applied scientists until twenty or thirty years ago, when a considerable attention started to be paid to systems governed by fractional differential equations (now commonly called fractional systems). All over the world a spreading scientific community has indeed brought to light that many real physical systems (systems with long memory or hereditary behavior) are well characterized by fractional differential equations. Fractional differentiation and fractional systems play now a very important role in various fields such as biology, bio-physics, control theory, economics, electrical engineering, electronics, electromagnetism, electrochemistry, image and signal processing, mechanics, mechatronics, physics, rheology, material modeling and thermal engineering.

In this context, the first IFAC workshop on Fractional Differentiation and its Applications, FDA'04, was held in Bordeaux, France, in 2004. This workshop aimed at bringing together experts in the field of fractional differentiation and its applications and all interested researchers, from universities and industries, to look at the state of the art and current research lines in theory, methodology, applications and tools.

This book integrates these parts gathering a selection of articles presented during FDA'04. Its attempt is to give to the reader a presentation of current research and the latest industrial applications of fractional differentiation. The first part is dedicated to mathematical tools and geometrical and physical aspects. The second part presents applications in the domains of ecophysiology, mechatronics, material modeling, thermal systems, electronics and electrical systems. Finally, the third part presents applications in systems analysis, implementation and simulation, system identification and system control.

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## Fractional differentiation and its applications

**Fractional Differentiation**

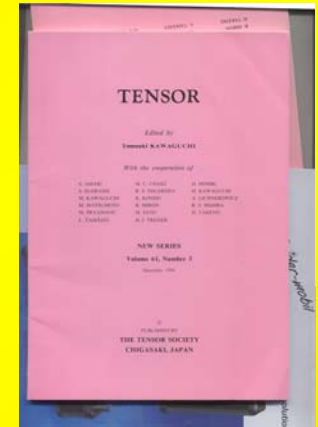
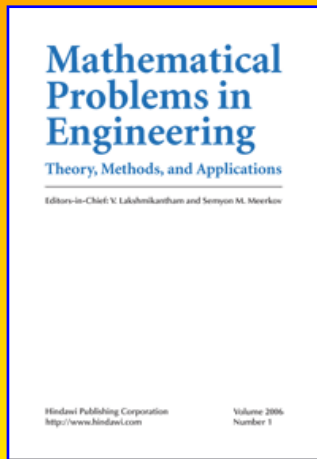
$$D_x^\alpha f(x) = \frac{1}{\Gamma(m-\alpha)} \left( \frac{d}{dx} \right)^m \int_a^x \frac{f(\tau)}{(x-\tau)^{\alpha-1}} d\tau$$

$$D_x^\alpha f(x) = \lim_{h \rightarrow 0} \frac{1}{h^\alpha} \sum_{k=0}^{\lfloor x/h \rfloor} (-1)^k \binom{\alpha}{k} f(x-kh)$$

$(X)(\omega) = (j\omega)^\alpha$

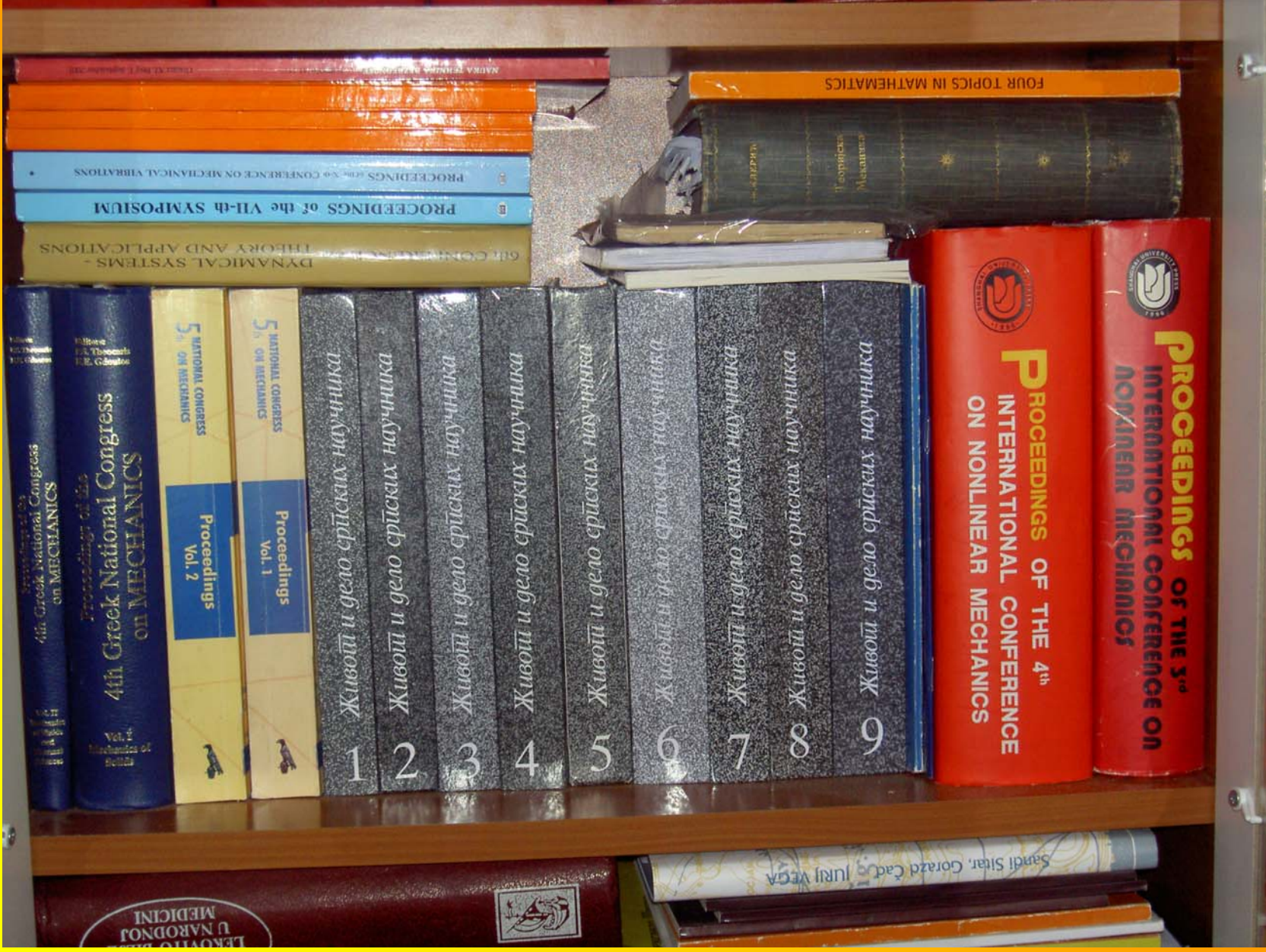
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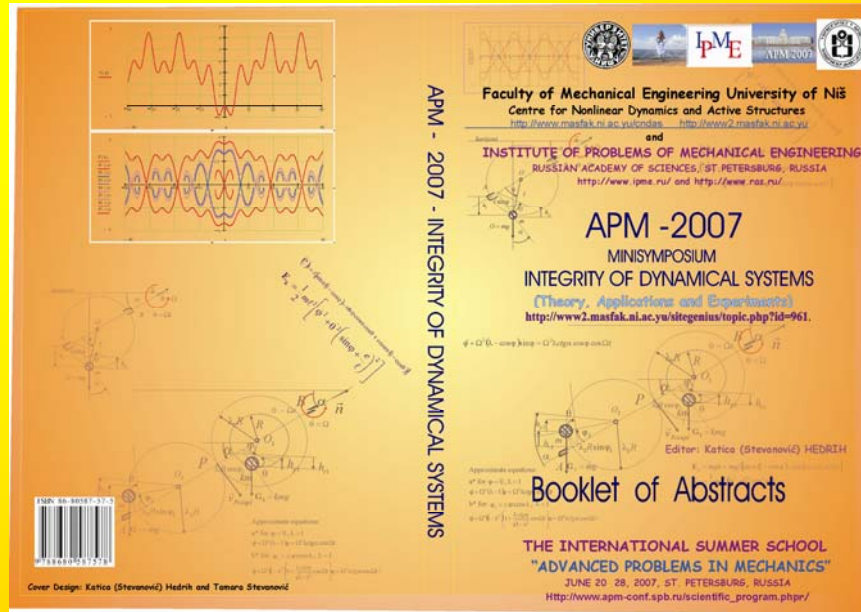
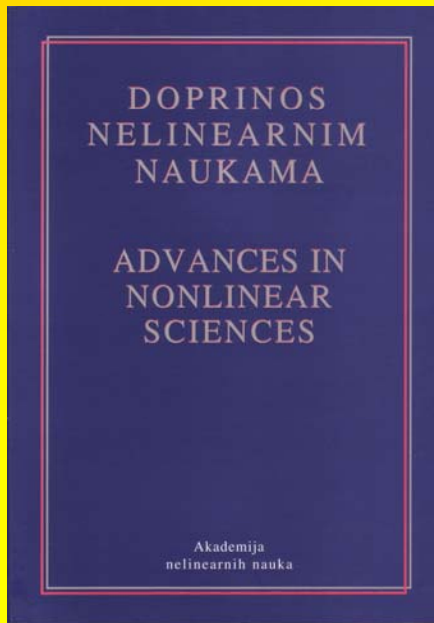
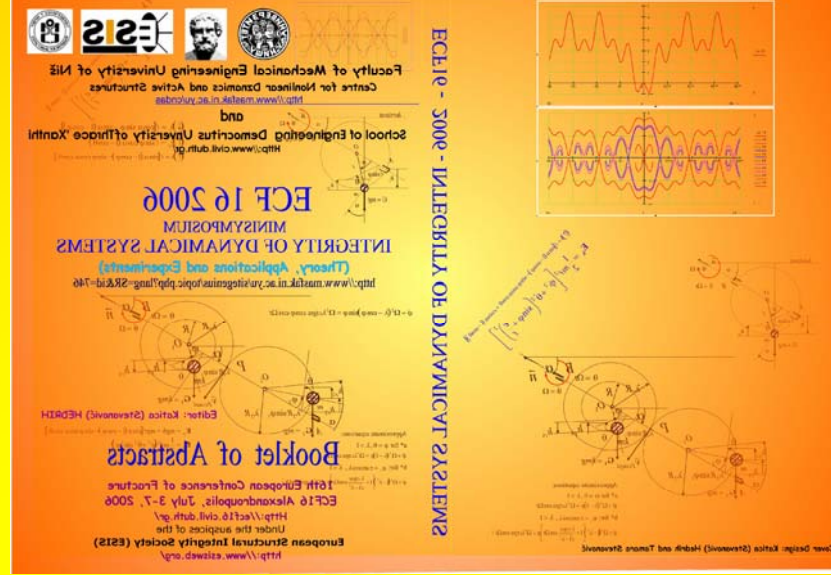
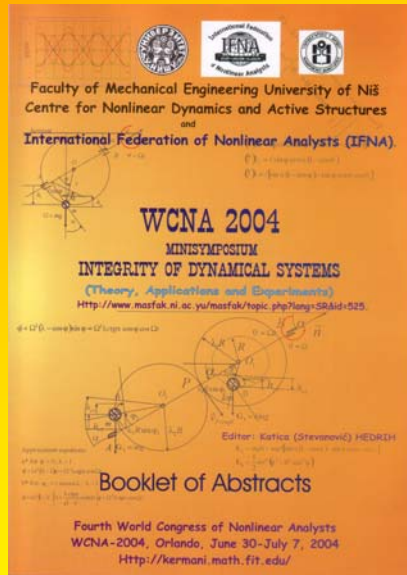
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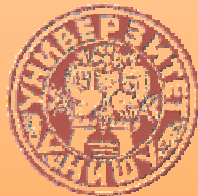
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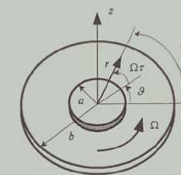


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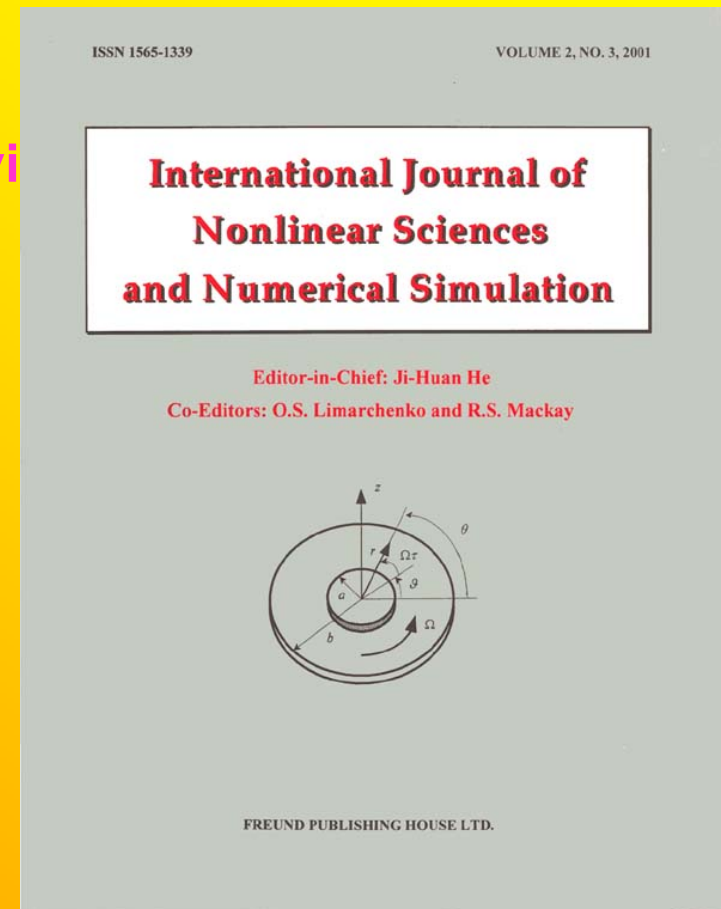
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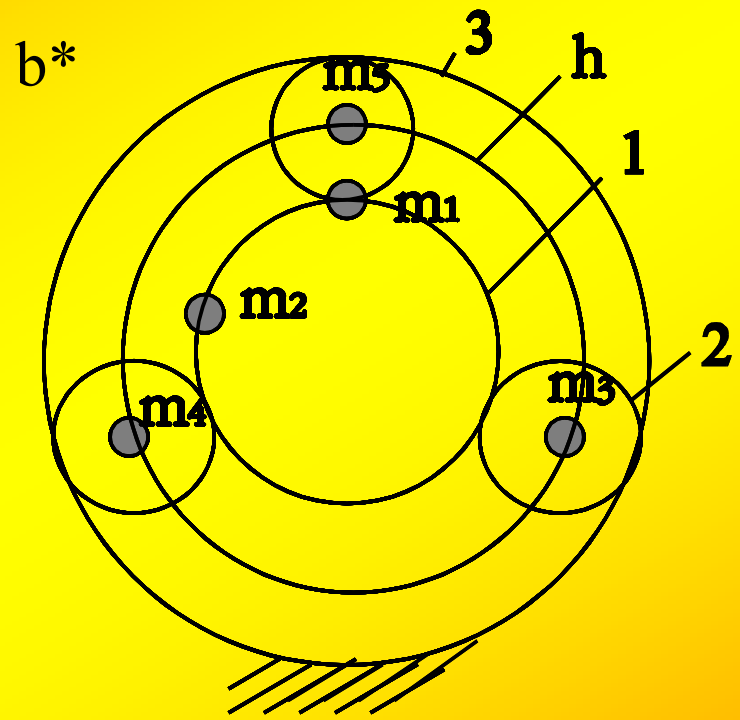
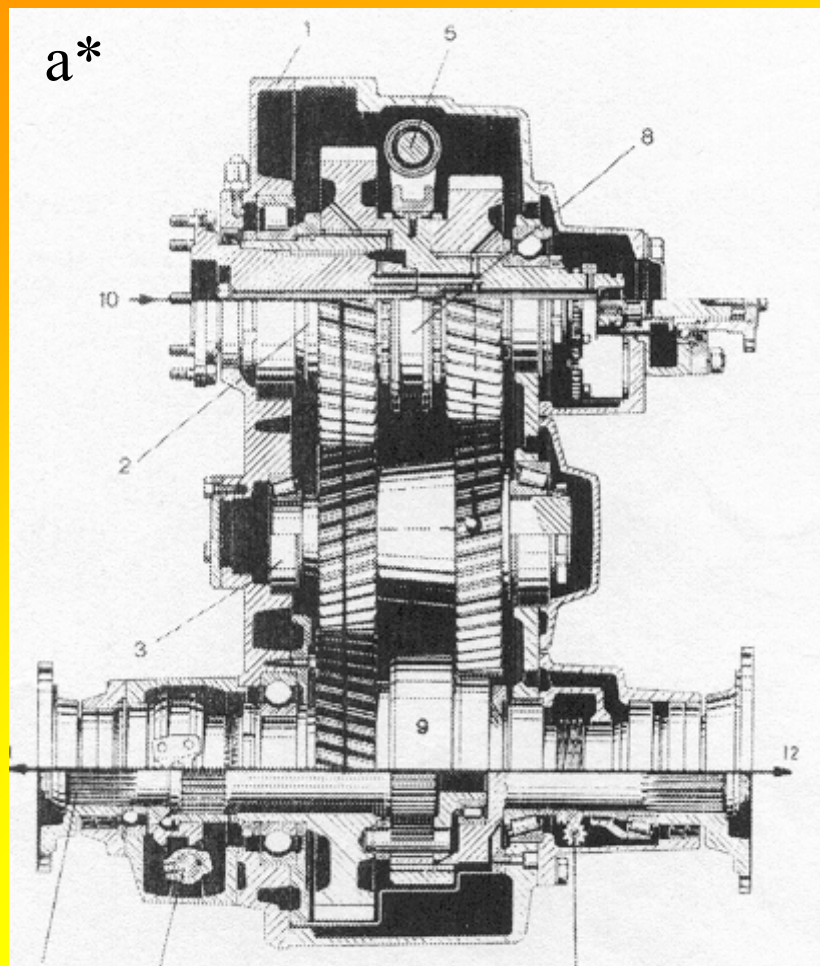
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**Figure 1.** Planetary reductor:  
**a\*** photograph of a real planetary reductor and  
**b\*** dynamic model of a planetary reductor



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