MA[INSKI FAKULTET UNIVERZITETA U NI[U
KATEDRA ZA MEHANIKU

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PISMENI DEO ISPITA IZ PREDMETA

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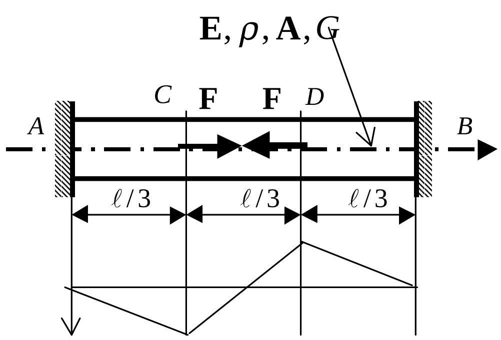
RE[ENJA

^ETVRTI ZADATAK:



Slika 4

Najve}e uzdu`no pomeranje koje je ostvareno pod dejstvom sile, u stanju stati~ke ravnote`e vratila, pod dejstvom uzdu`ne sile F, u ta~kama popre~nih preseka na udaljenju od po tre}ina raspona vratila mereno od ukle{tenja, je: $w_0 = \dfrac{Fl}{9EA}$.

Ovu vrednost pomeranja ta~aka preseka smo odredili zami{ljenim razdvajanjem (dekompozicijom) obostrano-ukle{tenog {tapa-vratila na dva konzolno ukle{tena vratila i jedno slobodno raspona po $\ell/3$, pri ~emu je jedna konzola optere}ena aksijalnom silom F_1 na istezanje, a druga aksijalnom silom F_4 takodje na istezanje, dok je slobodni {tap, izme|u njih optere}en na pritisak pritisak silama $F_2 = F_3$ istog intenziteta, a pri tome na osnovu uslova kompatibilnosti pomeranja njihova uzdu`na pomeranja slobodnih krajeva treba da budu ista. Zna~i da se radi o stati~ki neodredjenom sistemu u kome da bi smo odredili nepoznate otpore ukle{tenja i nepoznate deformacije dilatacije i pomeranja preseka potrebno je da prvo odredimo stati~ke nepoznate, a zatim ostale tra`ene parametre stanja diformacija napregnutog {tapa. Koriste}i uslove ravnote`e sila dobijamo slede}e:

$F_A - F_B + F - F = 0$ $\quad F_A = F_B = F_1 = F_4$ kao i $F_1 + F_2 = F$ i $F_3 + F_4 = F$

Korister}i uslove kompatibilnosti pomeranja preseka dobijamo:

$\Delta\ell_1 = \Delta\ell_2$ i $\Delta\ell_3 = \Delta\ell_4$

$\dfrac{F_1\left(\dfrac{\ell}{3}\right)}{EA} = \dfrac{1}{2}\dfrac{F_2\left(\dfrac{\ell}{3}\right)}{EA}$ i $\dfrac{F_4\left(\dfrac{\ell}{3}\right)}{EA} = \dfrac{1}{2}\dfrac{F_3\left(\dfrac{\ell}{3}\right)}{EA}$ odakle sledi da je: $2F_1 = F_2$ odnosno $2F_4 = F_3$

odnosno sledi da je:

$F_A = F_B = F_1 = F_4 = \dfrac{F}{3}$ i $F_3 = F_4 = \dfrac{2F}{3}$.

Sada pomeranja preseka C i D odredjujemo pmo}u odredjenih sila:

$$w_C = w_0 = +\frac{F_1\left(\frac{\ell}{3}\right)}{EA} = \frac{F\ell}{9EA} \qquad \text{i} \qquad w_D = -w_0 = -\frac{F_4\left(\frac{\ell}{3}\right)}{EA} = -\frac{F\ell}{9EA}$$

Zakon promene uzdu`nog pomeranja popre~nik preseka za ovako obostrano ukle{teno vratilo-{tap koje je napregnuto i deformisano parom suprotnih aksijalnih sila, prikazan je na slici br. 4, a analiti~ki se mo`e izraziti u obliku:

$$w = w(z,o) = f(z) = \begin{cases} 3\dfrac{w_0}{l}z; \quad 0 \le z \le \dfrac{l}{3} \\ -6\dfrac{w_0}{l}(z-\dfrac{\ell}{2}); \quad \dfrac{l}{3} \le z \le \dfrac{2l}{3} \\ 3\dfrac{w_0}{l}(z-\ell); \quad \dfrac{2l}{3} \le z \le l \end{cases}$$

gde je z uzdu`na koordinata u pravcu ose vratila, mereno od levog ukle{tenja. Prethodni analiti~ki izraz istovremeno predstavlja jedan od po~etnih uslova, kojim su pobu|ene uzdu`ne oscilacije u {tapu, po prestanku dejstva para sila. Drugi po~etni uslov je da su sve ta~ke {tapa u po~etnom trenutku bile u miru, pa je $\dfrac{\partial w(z,0)}{\partial t} = \varphi(z) = 0$.

Parcijalna diferencijalna jedna~ina longitudunalnih oscilacija homogenog {tapa je:

$$\frac{\partial^2 w(z,t)}{\partial t^2} = c^2\,\frac{\partial^2 w(z,t)}{\partial z^2}, \qquad c^2 = \frac{E}{\rho}, \qquad (1)$$

gde je $w(z,t)$ uzdu`no pomeranje du` ose z {tapa, a t vreme. Re{enje prethodne parcijalne diferencijalne jedna~ine uzdu`nih (longitudinalnih) oscilacija vratila-{tapa predpostavljamo u obliku:

w(z,t)= $\sum_{n=1}^{n=\infty}$ sin($\dfrac{n\pi}{l}$z)[$A_n\cos\omega_n t+B_n\sin\omega_n t$] , u kome je $\omega_n = \dfrac{n\pi}{\ell}\sqrt{\dfrac{E}{\rho}} = \dfrac{n\pi}{\ell}\sqrt{\dfrac{2(1+\mu)G}{\rho}}$

gde je $Z_n(z) = \sin(\dfrac{n\pi}{l}z)$ sopstvena, ortogonalna funkcija za obostrano ukle{ten {tap (vidi ud`benik *D. Ra{kovi}: Teorija oscilacija*).

U po~etnom trenutku za t=0 iz pretpostavljenog re{enja i njegovog izvoda po vremenu sledi da je:

$$w(z,0)= \sum_{n=1}^{\infty} A_n \sin\frac{n\pi}{l} z = f(z)$$

$$\dot{w}(z,0)= \sum_{n=1}^{\infty} \omega_n B_n \sin\frac{n\pi}{l} z = 0,$$

jer su po~etni uslovi su zadati u obliku:

$$w = w(z,o) = f(z) = \begin{cases} 3\dfrac{w_0}{l}z; \quad 0 \le z \le \dfrac{l}{3} \\ -6\dfrac{w_0}{l}(z-\dfrac{\ell}{2}); \quad \dfrac{l}{3} \le z \le \dfrac{2l}{3} \\ 3\dfrac{w_0}{l}(z-\ell); \quad \dfrac{2l}{3} \le z \le l \end{cases} \qquad \frac{\partial w(z,0)}{\partial t} = \varphi(z) = 0$$

pa sledi da su svi koeficijenti B_n=0 jednaki nuli; Koeficijente A_n , $n = 1,2,3,4,5,.....\infty$ odredjujemo pomo}u slede}ih izraza:

$$A_n = \frac{\int_0^l f(z)\sin\frac{n\pi}{l} z dz}{\int_0^l \sin^2\frac{n\pi}{l} z dz} = \frac{2}{l}\,[\int_0^{\frac{l}{3}} \frac{3w_0}{l} z \sin\frac{n\pi}{l} z dz + \int_{\frac{l}{3}}^{\frac{2l}{3}} \frac{6w_0}{l}(z-\frac{\ell}{2})\sin\frac{n\pi}{l} z dz + \int_{\frac{2l}{3}}^{l} \frac{3w_0}{l}(z-\ell)\sin\frac{n\pi}{l} z dz\,] \Rightarrow$$

$$A_n = \frac{4Fl}{EA}\frac{1}{n^2\pi^2}\cos\frac{n\pi}{2}\sin\frac{n\pi}{6} \;,$$

Kako je $\cos\frac{n\pi}{2}$ je jednak nuli za $n = 2p+1$, a $(-1)^p$ za $n = 2p$. odnosno:

$$\cos\frac{n\pi}{2} = \begin{cases} 0 & za \quad n = 2p+1 \\ \cos p\pi = (-1)^p & za \quad n = 2p \end{cases}$$

pa sledi da je:

$$A_{2p} = \frac{Fl}{EA}\frac{(-1)^p}{p^2\pi^2}\sin\frac{p\pi}{3},$$

Analizom vrednosti poslednjeg izraza za koeficijente razvoja re{enja po sopstvenim vrednostima dolazimo do slede}ih zaklju~aka:

$$A_{2(3k+1)} = -\frac{Fl\sqrt{3}}{2EA}\frac{1}{(3k+1)^2\pi^2}, \qquad za \qquad p = 3k+1$$

$$A_{2(3k+2)} = \frac{Fl\sqrt{3}}{2EA}\frac{1}{(3k+2)^2\pi^2}, \qquad za \qquad p = 3k+2$$

kao i da su $A_6 = 0$, $A_{12} = 0$, $A_{18} = 0$, $A_{24} = 0$ .... i tako dalje.

pomo}u odredjenig nepoznatih koeficijenata razvoja re{enja po sopstvenim funkcijama (oblicima oscilovanja), zakon uzdu`nih (longitudinalnih) oscilacija za zadate po~etne uslove, proizi{le iz prethodne deformacije {tapa, mo`emo napisati u obliku superpozicije vi{e sopstvenih harmonika (oblika) oscilovanja:

$$w(z,t) = \frac{Fl\sqrt{3}}{2EA\pi^2}\left\{\sum_{k=0}^{\infty}\frac{1}{(3k+2)^2}\sin\frac{2(3k+2)\pi z}{l}\cos\frac{2(3k+2)\pi}{l}t\sqrt{\frac{E}{\rho}} - \sum_{k=0}^{\infty}\frac{1}{(3k+1)^2}\sin\frac{2(3k+1)\pi z}{l}\cos\frac{2(3k+1)\pi}{l}t\sqrt{\frac{E}{\rho}}\right\}$$

b* Za zadate numeri~ke vrednosti: $\ell = 3[m]$, $E = 2\cdot 10^4\left[\frac{kN}{cm^2}\right]$ $\rho = 7{,}85\cdot 10^3\left[\frac{kg}{m^3}\right]$, najni`a kru`na frekvencija longitudinalnih oscilacija obostranoukle{tenog {tapa, kru`nog popre~nog preseka, kojim mo`e da osciluje u op{tem slu~aju je:

$$\omega_1 = \frac{\pi}{\ell}\sqrt{\frac{E}{\rho}} = \frac{\pi}{3[m]}\sqrt{\frac{2\cdot 10^4\left[\frac{kN}{cm^2}\right]}{7{,}85\cdot 10^3\left[\frac{kg}{m^3}\right]}} = 5285{,}5\left[\sec^{-1}\right]$$

za zadate po~etne uslove, najni`a kru`na frekvencija kojom vratilo osciluje je:

$$\omega_2 = \frac{2\pi}{\ell}\sqrt{\frac{E}{\rho}} = \frac{2\pi}{3[m]}\sqrt{\frac{2\cdot 10^4\left[\frac{kN}{cm^2}\right]}{7{,}85\cdot 10^3\left[\frac{kg}{m^3}\right]}} = 10571\left[\sec^{-1}\right]$$

dok su ostale, vi{e kru`ne frekvencije kojim vratilo za zadate po~etne uslove longitudinalno osciluje:

$$\omega_4 = \frac{4\pi}{\ell}\sqrt{\frac{E}{\rho}} = \frac{4\pi}{3[m]}\sqrt{\frac{2\cdot 10^4\left[\frac{kN}{cm^2}\right]}{7{,}85\cdot 10^3\left[\frac{kg}{m^3}\right]}} = 21143\left[\sec^{-1}\right]$$

$$\omega_8 = \frac{8\pi}{\ell}\sqrt{\frac{E}{\rho}} = \frac{8\pi}{3[m]}\sqrt{\frac{2\cdot 10^4\left[\frac{kN}{cm^2}\right]}{7{,}85\cdot 10^3\left[\frac{kg}{m^3}\right]}} = 42286\left[\sec^{-1}\right]$$

$$\omega_{10} = \frac{10\pi}{\ell}\sqrt{\frac{E}{\rho}} = 52855\left[\sec^{-1}\right]$$

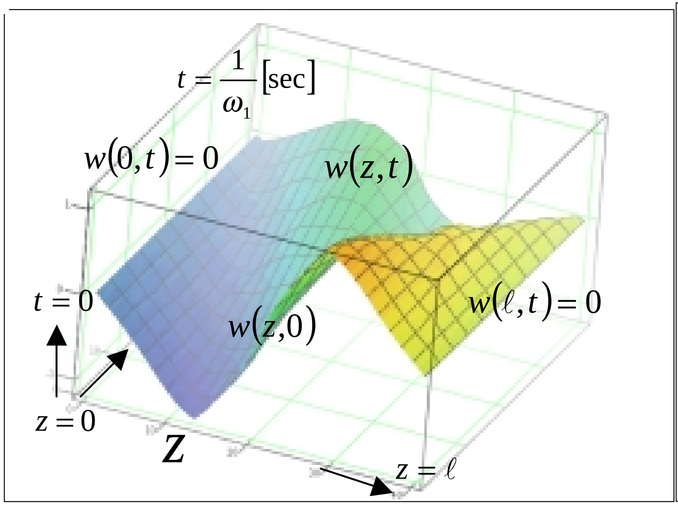
$$\omega_{14}=\frac{14\pi}{\ell}\sqrt{\frac{E}{\rho}}=73997\left[\sec^{-1}\right]$$

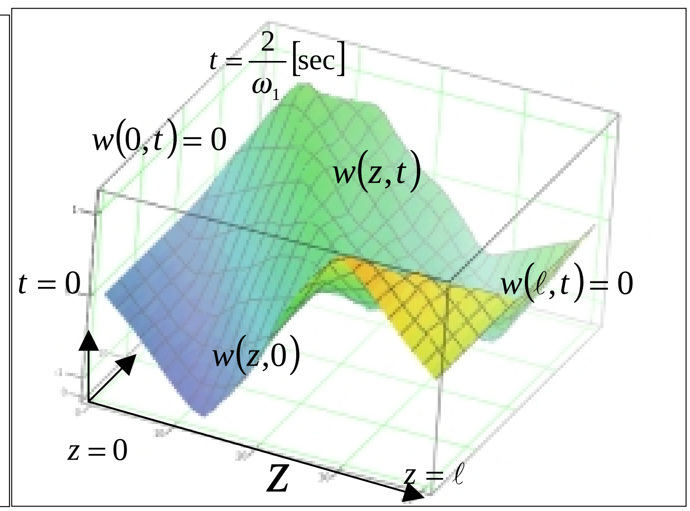
itd......

$$\omega_{2(3k+1)}=\frac{2(3k+1)\pi}{\ell}\sqrt{\frac{E}{\rho}}=10571(3k+1)\left[\sec^{-13k+1}\right]$$

i $$\omega_{2(3k+2)}=\frac{2(3k+2)\pi}{\ell}\sqrt{\frac{E}{\rho}}=10571(3k+2)\left[\sec^{-13k+1}\right]$$

Na slikama koje slede prikazane su povr{i promene uzdu`nih elongacija $w(z,t)$ popre~nih preseka u funkciji plo`aja preseka z od levog ukle{tenja za $z=0$ pa preko raspona do desnog ukle{enja za $z=\ell$ i trenutka vremena t za intervale vremena merene recipro~nim vrednostima kru`ne frekvencije $\frac{n}{\omega_1}$, $n=1,2,3...$ .Sa povr{i se uo~avaju i grani~ni $w(0,t)=0$ i $w(0,\ell)=0$ i po~etni uslovi $w(z,0)=f(z)$, i $\frac{\partial w(z,0)}{\partial t}=\varphi(z)=0$ kako je zadato na{im zadatkom.





M

$N := 40 \quad i := 0..N \quad j := 0..N$

$x_i := 0 + 0.0783\cdot i \quad y_j := 0 + 0.025\cdot j$

$$f(x,y) := \sum_{k=0}^{40}\left[\left[\sin[2\cdot(2\cdot k+2)\cdot x]\cdot\frac{\cos[2\cdot(3\cdot k+2)y]}{(3\cdot k+2)^2}\right]-\sin[2\cdot(3\cdot k+1)\cdot x]\cdot\frac{\cos[2\cdot(3\cdot k+1)\cdot y]}{(3\cdot k+1)^2}\right]$$

$M_{i,j} := f(x_i,y_j)$

M

$N := 40 \quad i := 0..N \quad j := 0..N$

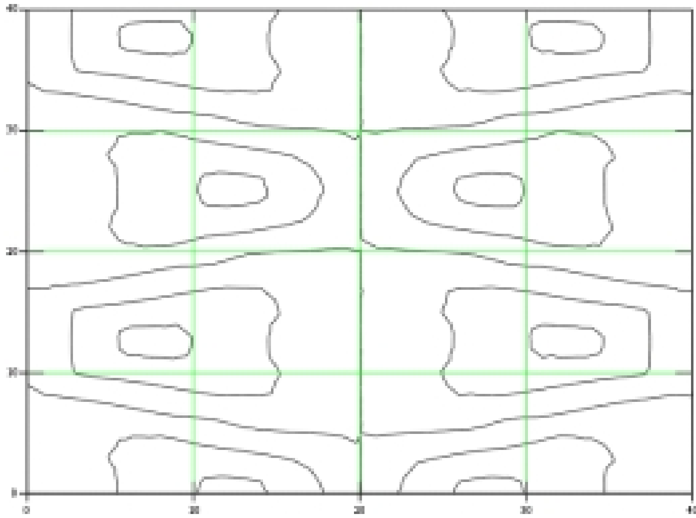
z je vreme t je 2 sekunde

$x_i := 0 + 0.0783\cdot i \quad y_j := 0 + 0.05\cdot j$

x je kordinata duz stapa i menja se od nule do pi

$$f(x,y) := \sum_{k=0}^{40}\left[\left[\sin[2\cdot(2\cdot k+2)\cdot x]\cdot\frac{\cos[2\cdot(3\cdot k+2)y]}{(3\cdot k+2)^2}\right]-\sin[2\cdot(3\cdot k+1)\cdot x]\cdot\frac{\cos[2\cdot(3\cdot k+1)\cdot y]}{(3\cdot k+1)^2}\right]$$

$M_{i,j} := f(x_i,y_j)$



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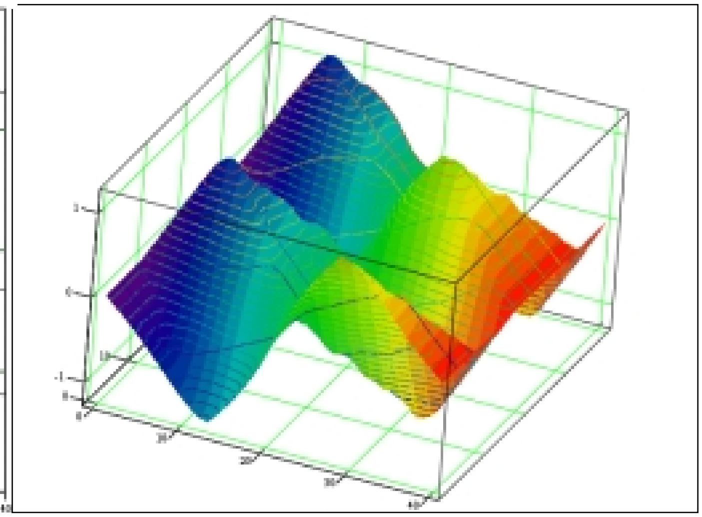


M

$N := 40 \qquad i := 0..N \qquad j := 0..N$

$x_i := 0 + 0.0783 \cdot i \qquad y_j := 0 + 0.125 \cdot j$

z je vreme   t je 5 sekundi

x je kordinata duz stapa i menja se od nule do pi

$$f(x,y) := \sum_{k=0}^{40} \left[\left[\sin[2\cdot(2\cdot k+2)\cdot x]\cdot\frac{\cos[2\cdot(3\cdot k+2)y]}{(3\cdot k+2)^2} \right] - \sin[2\cdot(3\cdot k+1)\cdot x]\cdot\frac{\cos[2\cdot(3\cdot k+1)\cdot y]}{(3\cdot k+1)^2} \right]$$

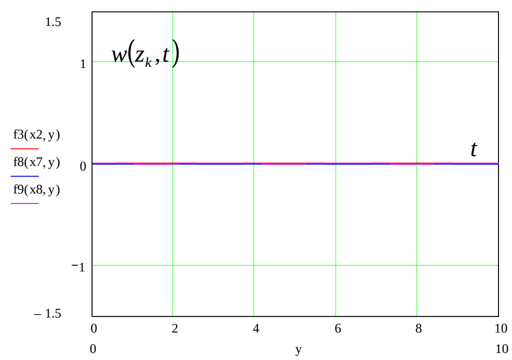
$M_{i,j} := f(x_i, y_j)$

Preseci koji prinudno miruju u svakom trenutku vremena, za zadate po~etne uslove: $z = 0;\ \frac{\ell}{2};\ \ell$, to su preseci u ule{tenjima i na sredini vratila.

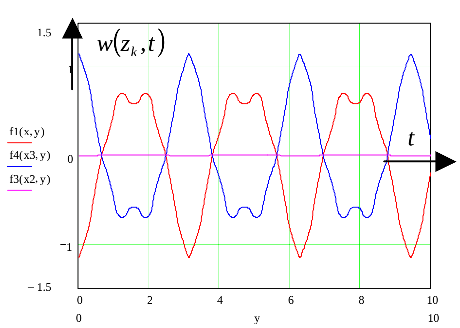
$x7 := 0 \qquad f8(x7,y) := \sum_{k=0}^{10} \left[\left[\sin[2\cdot(2\cdot k+2)\cdot x7]\cdot\frac{\cos[2\cdot(3\cdot k+2)y]}{(3\cdot k+2)^2} \right] - \sin[2\cdot(3\cdot k+1)\cdot x7]\cdot\frac{\cos[2\cdot(3\cdot k+1)\cdot y]}{(3\cdot k+1)^2} \right]$

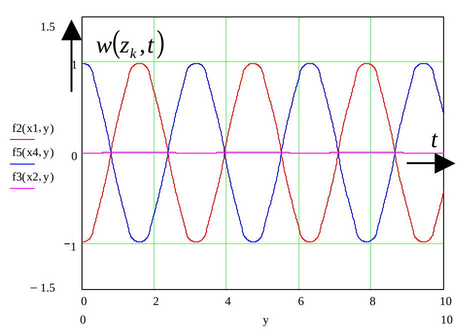
$x8 := 3.14 \qquad f9(x8,y) := \sum_{k=0}^{10} \left[\left[\sin[2\cdot(2\cdot k+2)\cdot x8]\cdot\frac{\cos[2\cdot(3\cdot k+2)y]}{(3\cdot k+2)^2} \right] - \sin[2\cdot(3\cdot k+1)\cdot x8]\cdot\frac{\cos[2\cdot(3\cdot k+1)\cdot y]}{(3\cdot k+1)^2} \right]$

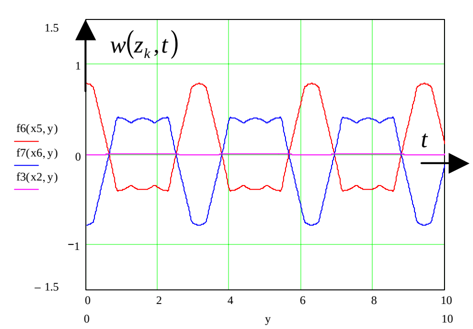
$x2 := \frac{3.14}{2} \qquad f3(x2,y) := \sum_{k=0}^{10} \left[\left[\sin[2\cdot(2\cdot k+2)\cdot x2]\cdot\frac{\cos[2\cdot(3\cdot k+2)y]}{(3\cdot k+2)^2} \right] - \sin[2\cdot(3\cdot k+1)\cdot x2]\cdot\frac{\cos[2\cdot(3\cdot k+1)\cdot y]}{(3\cdot k+1)^2} \right]$

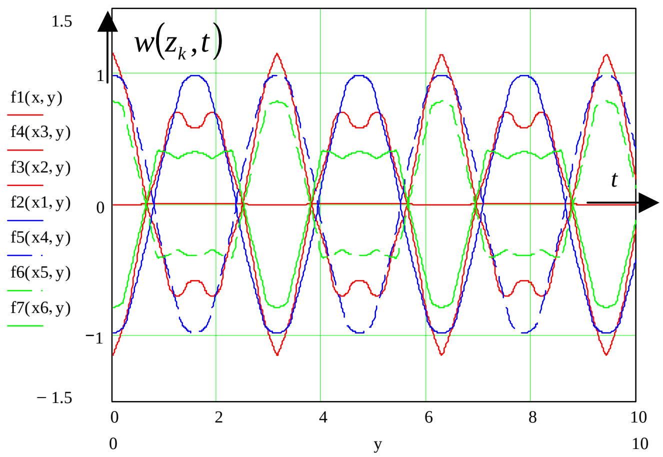


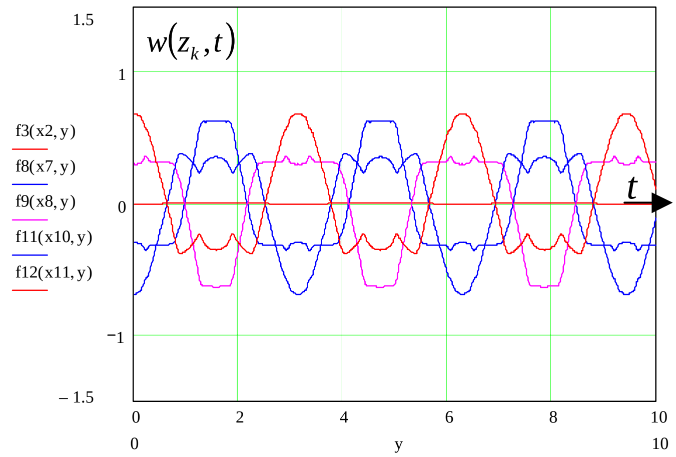
Preseci simetri~no rasporedjeni u odnosu na ravan simetrije kroz presek za $z = \frac{\ell}{2}$, osciluju u suprotnim smerovima, {to se lepo mo`e uo~ili sa slede}ih kinemati~kih dijagrama longitdinalnih oscilacija: za $z = \frac{\ell}{3};\ \frac{2\ell}{3}$, zatim za $z = \frac{\ell}{4};\ \frac{3\ell}{4}$, zatim za $z = \frac{2\ell}{5};\ \frac{3\ell}{5}$ i tako dalje....











z je vreme t je 10 sekundi x je kordinata duz stapa i menja se od nule do pi

$$x := \frac{3.14}{3} \quad f1(x,y) := \sum_{k=0}^{10}\left[\left[\sin[2\cdot(2\cdot k+2)\cdot x]\cdot\frac{\cos[2\cdot(3\cdot k+2)y]}{(3\cdot k+2)^2}\right]-\sin[2\cdot(3\cdot k+1)\cdot x]\cdot\frac{\cos[2\cdot(3\cdot k+1)\cdot y]}{(3\cdot k+1)^2}\right]$$

$$x1 := \frac{3.14}{4} \quad f2(x1,y) := \sum_{k=0}^{10}\left[\left[\sin[2\cdot(2\cdot k+2)\cdot x1]\cdot\frac{\cos[2\cdot(3\cdot k+2)y]}{(3\cdot k+2)^2}\right]-\sin[2\cdot(3\cdot k+1)\cdot x1]\cdot\frac{\cos[2\cdot(3\cdot k+1)\cdot y]}{(3\cdot k+1)^2}\right]$$

$$x2 := \frac{3.14}{2} \quad f3(x2,y) := \sum_{k=0}^{10}\left[\left[\sin[2\cdot(2\cdot k+2)\cdot x2]\cdot\frac{\cos[2\cdot(3\cdot k+2)y]}{(3\cdot k+2)^2}\right]-\sin[2\cdot(3\cdot k+1)\cdot x2]\cdot\frac{\cos[2\cdot(3\cdot k+1)\cdot y]}{(3\cdot k+1)^2}\right]$$

$$x7 := \frac{3.14}{12} \quad f8(x7,y) := \sum_{k=0}^{10}\left[\left[\sin[2\cdot(2\cdot k+2)\cdot x7]\cdot\frac{\cos[2\cdot(3\cdot k+2)y]}{(3\cdot k+2)^2}\right]-\sin[2\cdot(3\cdot k+1)\cdot x7]\cdot\frac{\cos[2\cdot(3\cdot k+1)\cdot y]}{(3\cdot k+1)^2}\right]$$

$$x3 := \frac{6.28}{3} \quad f4(x3,y) := \sum_{k=0}^{10}\left[\left[\sin[2\cdot(2\cdot k+2)\cdot x3]\cdot\frac{\cos[2\cdot(3\cdot k+2)y]}{(3\cdot k+2)^2}\right]-\sin[2\cdot(3\cdot k+1)\cdot x3]\cdot\frac{\cos[2\cdot(3\cdot k+1)\cdot y]}{(3\cdot k+1)^2}\right]$$

$$x4 := \frac{3\cdot 3.14}{4} \quad f5(x4,y) := \sum_{k=0}^{10}\left[\left[\sin[2\cdot(2\cdot k+2)\cdot x4]\cdot\frac{\cos[2\cdot(3\cdot k+2)y]}{(3\cdot k+2)^2}\right]-\sin[2\cdot(3\cdot k+1)\cdot x4]\cdot\frac{\cos[2\cdot(3\cdot k+1)\cdot y]}{(3\cdot k+1)^2}\right]$$

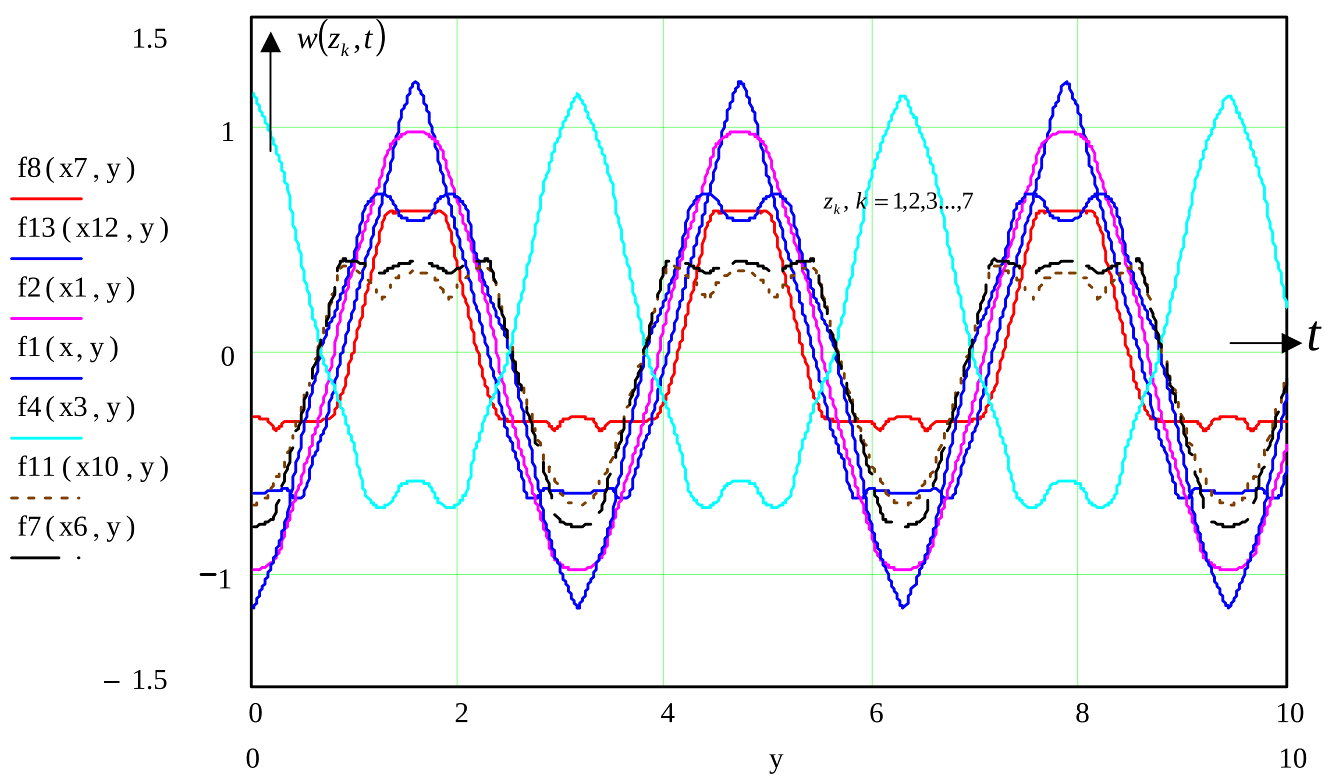
$$x8 := 3.14\cdot\frac{11}{12} \quad f9(x8,y) := \sum_{k=0}^{10}\left[\left[\sin[2\cdot(2\cdot k+2)\cdot x8]\cdot\frac{\cos[2\cdot(3\cdot k+2)y]}{(3\cdot k+2)^2}\right]-\sin[2\cdot(3\cdot k+1)\cdot x8]\cdot\frac{\cos[2\cdot(3\cdot k+1)\cdot y]}{(3\cdot k+1)^2}\right]$$

$$x5 := \frac{3\cdot 3.14}{5} \quad f6(x5,y) := \sum_{k=0}^{10}\left[\left[\sin[2\cdot(2\cdot k+2)\cdot x5]\cdot\frac{\cos[2\cdot(3\cdot k+2)y]}{(3\cdot k+2)^2}\right]-\sin[2\cdot(3\cdot k+1)\cdot x5]\cdot\frac{\cos[2\cdot(3\cdot k+1)\cdot y]}{(3\cdot k+1)^2}\right]$$

$$x10 := \frac{3.14\cdot 5}{12} \quad f11(x10,y) := \sum_{k=0}^{10}\left[\left[\sin[2\cdot(2\cdot k+2)\cdot x10]\cdot\frac{\cos[2\cdot(3\cdot k+2)y]}{(3\cdot k+2)^2}\right]-\sin[2\cdot(3\cdot k+1)\cdot x10]\cdot\frac{\cos[2\cdot(3\cdot k+1)\cdot y]}{(3\cdot k+1)^2}\right]$$

$$x6 := \frac{2\cdot 3.14}{5} \quad f7(x6,y) := \sum_{k=0}^{10}\left[\left[\sin[2\cdot(2\cdot k+2)\cdot x6]\cdot\frac{\cos[2\cdot(3\cdot k+2)y]}{(3\cdot k+2)^2}\right]-\sin[2\cdot(3\cdot k+1)\cdot x6]\cdot\frac{\cos[2\cdot(3\cdot k+1)\cdot y]}{(3\cdot k+1)^2}\right]$$

$$x11 := \frac{3.14\cdot 7}{12} \quad f12(x11,y) := \sum_{k=0}^{10}\left[\left[\sin[2\cdot(2\cdot k+2)\cdot x11]\cdot\frac{\cos[2\cdot(3\cdot k+2)y]}{(3\cdot k+2)^2}\right]-\sin[2\cdot(3\cdot k+1)\cdot x11]\cdot\frac{\cos[2\cdot(3\cdot k+1)\cdot y]}{(3\cdot k+1)^2}\right]$$



Longitudinalne elongacije preseka jedne polovine vratila na rastojanjima od $z=\frac{\ell}{12};\ \frac{\ell}{6};\ \frac{\ell}{4};\ \frac{\ell}{3};\ \frac{2\ell}{5};\ \frac{5\ell}{12};$ u funkciji vremena

$$x7 := \frac{3.14}{12} \quad f8(x7,y) := \sum_{k=0}^{10} \left[\left[\sin[2\cdot(2\cdot k+2)\cdot x7]\cdot\frac{\cos[2\cdot(3\cdot k+2)y]}{(3\cdot k+2)^2} \right] - \sin[2\cdot(3\cdot k+1)\cdot x7]\cdot\frac{\cos[2\cdot(3\cdot k+1)\cdot y]}{(3\cdot k+1)^2} \right]$$

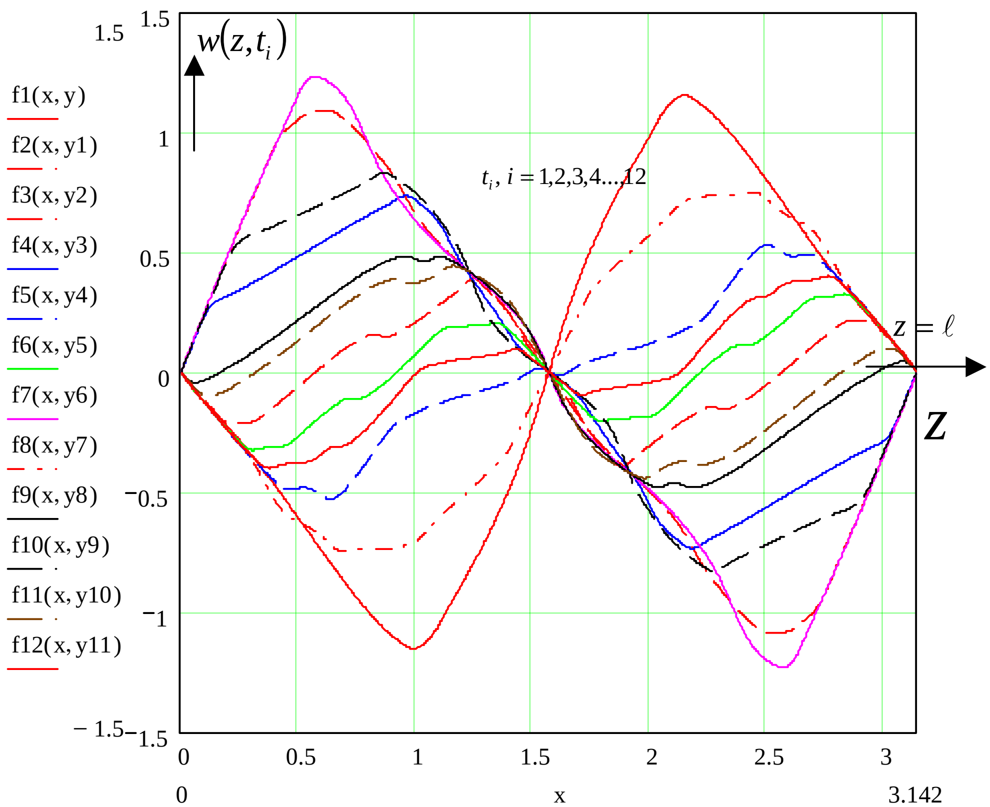
$$x8 := 3.14\cdot\frac{11}{12} \quad f9(x8,y) := \sum_{k=0}^{10} \left[\left[\sin[2\cdot(2\cdot k+2)\cdot x8]\cdot\frac{\cos[2\cdot(3\cdot k+2)y]}{(3\cdot k+2)^2} \right] - \sin[2\cdot(3\cdot k+1)\cdot x8]\cdot\frac{\cos[2\cdot(3\cdot k+1)\cdot y]}{(3\cdot k+1)^2} \right]$$

$$x10 := \frac{3.14\cdot 5}{12} \quad f11(x10,y) := \sum_{k=0}^{10} \left[\left[\sin[2\cdot(2\cdot k+2)\cdot x10]\cdot\frac{\cos[2\cdot(3\cdot k+2)y]}{(3\cdot k+2)^2} \right] - \sin[2\cdot(3\cdot k+1)\cdot x10]\cdot\frac{\cos[2\cdot(3\cdot k+1)\cdot y]}{(3\cdot k+1)^2} \right]$$

$$x11 := \frac{3.14\cdot 7}{12} \quad f12(x11,y) := \sum_{k=0}^{10} \left[\left[\sin[2\cdot(2\cdot k+2)\cdot x11]\cdot\frac{\cos[2\cdot(3\cdot k+2)y]}{(3\cdot k+2)^2} \right] - \sin[2\cdot(3\cdot k+1)\cdot x11]\cdot\frac{\cos[2\cdot(3\cdot k+1)\cdot y]}{(3\cdot k+1)^2} \right]$$

$$x12 := \frac{3.14}{6} \quad f13(x12,y) := \sum_{k=0}^{10} \left[\left[\sin[2\cdot(2\cdot k+2)\cdot x12]\cdot\frac{\cos[2\cdot(3\cdot k+2)y]}{(3\cdot k+2)^2} \right] - \sin[2\cdot(3\cdot k+1)\cdot x12]\cdot\frac{\cos[2\cdot(3\cdot k+1)\cdot y]}{(3\cdot k+1)^2} \right]$$

$$x13 := \frac{3.14\cdot 5}{6} \quad f14(x13,y) := \sum_{k=0}^{10} \left[\left[\sin[2\cdot(2\cdot k+2)\cdot x13]\cdot\frac{\cos[2\cdot(3\cdot k+2)y]}{(3\cdot k+2)^2} \right] - \sin[2\cdot(3\cdot k+1)\cdot x13]\cdot\frac{\cos[2\cdot(3\cdot k+1)\cdot y]}{(3\cdot k+1)^2} \right]$$



$$y := 628\cdot\frac{5285}{52855}$$

$$y1 := 628\cdot\frac{30000}{52855}$$

$$y2 := 628\cdot\frac{15000}{52855}$$

$$y3 := 628\cdot\frac{7500}{52855}$$

$$y4 := 628\cdot\frac{12642}{52855}$$

$$y5 := 628\cdot\frac{11961}{52855}$$

$$y6 := 628\cdot\frac{11500}{52855}$$

$$y7 := 628\cdot\frac{11400}{52855}$$

Elongacije preseka u diskretnim trenucima vremena .